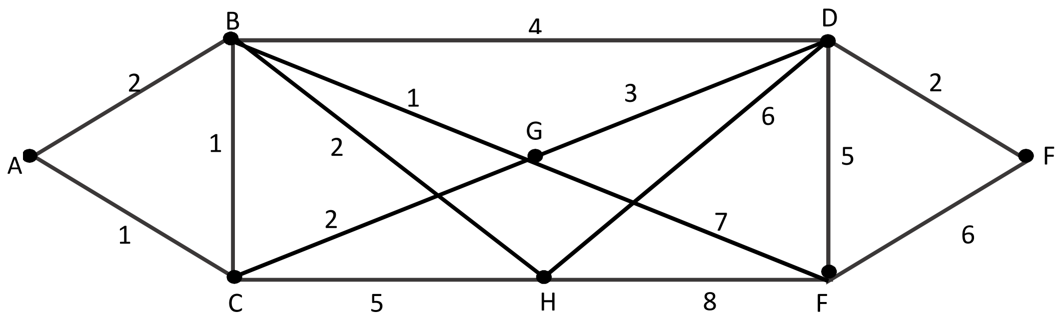
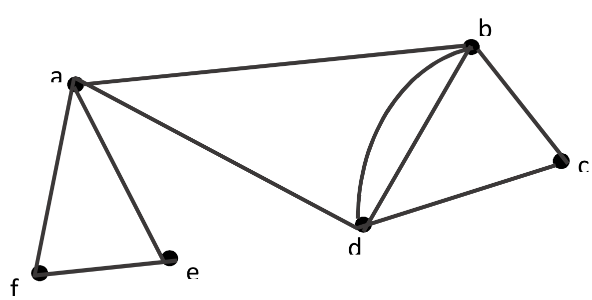
Name of the Course	**: B.Sc. (H) Mathematics**
Unique Paper Code	**: 32357505**
Name of the Paper	**: DSE-II Discrete Mathematics**
Semester	**: V Semester**
Duration	**: 3 hours**
Maximum Marks	**: 75 Marks**

Attempt any four questions. All questions carry equal marks.

1. Apply Dijkstra's Algorithm OR Improved version of Dijkstra's Algorithm to find a shortest path from A to F, also write steps wherever possible.

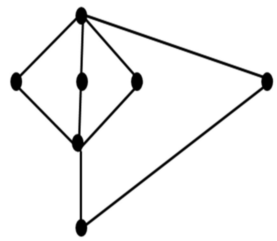


In the pseudograph given below either describe an Eulerian circuit or explain why no Eulerian circuit exists.



2. Prove or disprove the statement: *Homomorphic image of modular lattice is modular.*

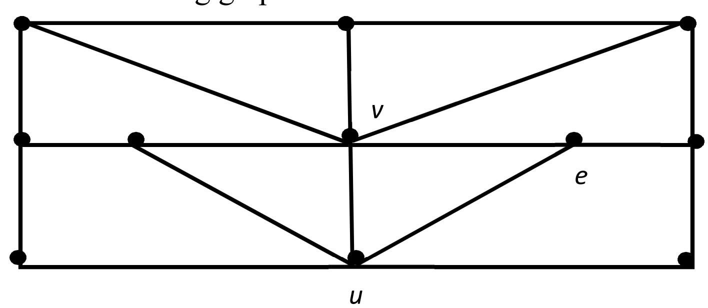
Construct a lattice L with 0 and 1, so that L has at least one element having three complements.



Verify whether the lattice given below is modular and /or distributive, by using M3-N5 theorem.

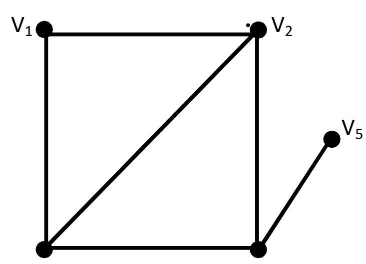
Find the disjunctive normal form of the Boolean polynomial $p = (xy' + xz)' + x'$. Further, find the conjunctive normal form of 'p'.

3. Explain the Königsberg bridge problem and discuss the solution provided by graph theory to this problem. The degree of each vertex of a certain graph is either 4 or 6. The graph has 12 vertices and 31 edges. How many vertices of degree 4 are there? Draw the subgraphs G $\backslash\{e\}$, G $\backslash\{v\}$ and G $\backslash\{u\}$ of the following graph G.

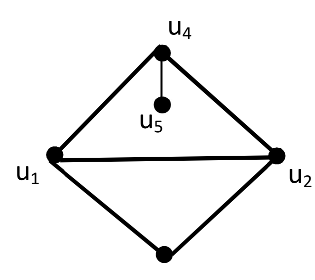


G

Find the adjacency matrices A_1 and A_2 of the graphs G_1 and G_2 shown below. Find a permutation matrix P such that $A_2 = PA_1P^T$.



$\mathbf{G_1}$



$\mathbf{G_2}$

4. Is the expression $y'z'$ an implicant of the expression $xy'z' + x'y + x'y'z' + x'yz$. Give reasons for your answer.

What are prime implicants of $p = xyz + xyz' + xy'z + x'yz + x'y'z$?

Using K-maps or Quine–McCluskey method, find the minimal sum of products form of the polynomial p.

Give the symbolic representation of the circuit $q = (x'yz)' + x'yz' + (xy'z)' + xy'z'$.

Also, draw the contact diagram of above circuit q.

5. Let $X = \{1, 2, 3\}$. Consider the partial ordered set $(L, \leq)$ where $L = P(X)$ is the power set of X and '$\leq$' is defined as, $U \leq V$ if and only if $U \subseteq V \quad \forall U, V \in L$. Draw Hasse diagram of $(L, \leq)$. Prove or disprove that $(L, \leq)$ is a chain. Justify your answer. Find a subset of $(L, \leq)$ that forms a chain with respect to the same partial order relation.

Consider poset $Q = \{a, b\}$ where $a < b$. Is the map $\theta: L \to Q$ order preserving where

$$\theta(U) = \begin{cases} a, & \text{if } U = X \\ b, & \text{if } U \neq X \end{cases}.$$

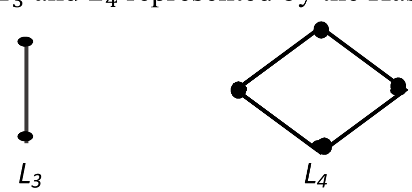
Justify your answer.

Exhibit an order isomorphism between the given partial ordered set $L = P(X)$ and partial ordered set S of all positive divisors of 30, with respect to the order that for any $a, b \in S$, $a \leq b$ if and only if a divides b. Are the Hasse diagrams of two partial ordered sets $(P(X), \subseteq)$ and $(S, \leq)$ identical?

State a result describing a relationship between the existence of an order isomorphic map between any two finite ordered sets A and B and their Hasse Diagrams. Can you prove this statement?

6. Let $L_1 = \{2, 4, 8, 10, 20, 40\}$ and $L_2 = \{1, 2, 4, 5, 20\}$ be partially ordered sets with divisibility as the partial order relation. Are L_1 and L_2 lattices? Justify your answer. Show that the collection of all subgroups of a group G forms a lattice.

Consider lattices L_3 and L_4 represented by the Hasse diagrams shown below



Draw the Hasse diagram of lattice $L_3 \times L_4$.

Give example of a subset S of a lattice L, which is not a sublattice of L but is itself is a lattice.

Name of Course	: **CBCS B.Sc. (H) Mathematics**
Unique Paper Code	: **32357504**
Name of Paper	: **DSE-II Mathematical Finance**
Semester	: **V**
Duration	: **3 hours**
Maximum Marks	: **75 Marks**

Attempt any four questions. All questions carry equal marks.

1. Determine the effective rates of interest corresponding to the 5% rate of interest compounded daily, weekly, monthly, quarterly, semi-annually, and continuously. Arrange them in increasing order.

2. If one-year and two-year spot rates are $s_1 = 4\%$ and $s_2 = 5\%$ respectively, then find the forward rate f_{12}. If the spot rate curve is $(4.0, 5.0, 5.4, 5.6, 5.8, 6.0)$, then find the spot rate curve of the next year.

3. Assume that there are three assets having mean rates of return $\bar{r}_1 = 8\%$, $\bar{r}_2 = 10\%$, $\bar{r}_3 = 6\%$, standard deviations $\sigma_1 = 1.5$, $\sigma_2 = 0.5$, $\sigma_3 = 1.2$ and correlations $\rho_{12} = 0.3$, $\rho_{23} = 0$, $\rho_{13} = -0.2$.

(a) Find the covariance matrix for these three assets.
(b) Find the minimum-variance portfolio.
(c) Find another efficient portfolio by setting $\lambda = 1$, $\mu = 0$.
(d) If the risk-free rate is $r_f = 5\%$, then find the one fund of risky assets as specified in one-fund theorem.

4. Consider a market in which there are only two risky assets *A* and *B*, and a risk-free asset *F*. The total values of the assets in the market are \$200,000 and \$100,000, respectively. The market portfolio *M* consists of assets *A* and *B* in proportion to their market values. The following information is known: $r_F = .07, \bar{r}_A = .12, \bar{r}_B = .15, \sigma_A^2 = .06, \sigma_B^2 = .09, \sigma_{AB} = .01$.

(a) Write down the expression for the market portfolio in terms of the weights w_1 and w_2 of the two assets *A* and *B* in the market portfolio.
(b) Find the expected return and the standard deviation of the market portfolio *M*.
(c) Calculate σ_{AM}, σ_{BM} and hence find the beta of the stocks *A* and *B*.
(d) Verify that the beta of the market portfolio *M* is 1.

5. Consider a long forward contract to purchase a non-dividend paying stock in 6 months. The current price of the stock is \$100 and the six-month risk-free rate is 10% per annum compounding continuously. Obtain the forward price and discuss the arbitrage opportunities if the forward price is \$104.

6. The price of a non-dividend paying stock is \$19 and the price of a three-month European call option on the above stock with strike price \$20 is \$1. The risk-free rate is 4% per annum compounding continuously. Find the price of a 3-month European put option written on the same stock with strike price \$20? If the above European put option is selling at \$ 2, then identify an arbitrage opportunity if it exists.

Name of the Course	**: B.Sc. (H) Mathematics**
Unique Paper Code	**: 32357505**
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Semester	**: V Semester**
Duration	**: 3 hours**
Maximum Marks	**: 75 Marks**

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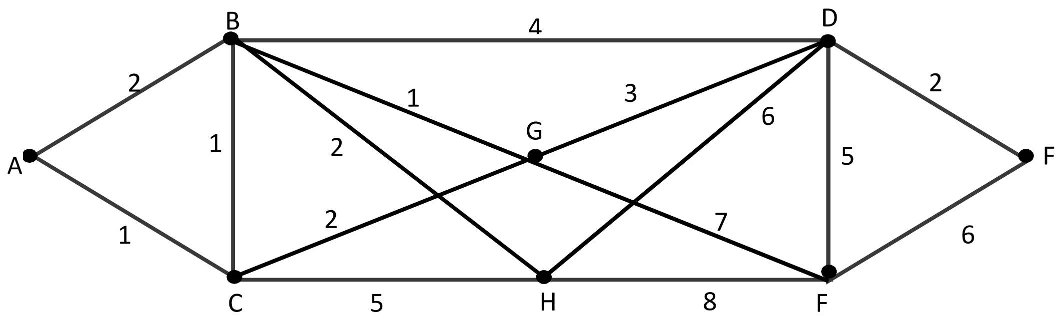

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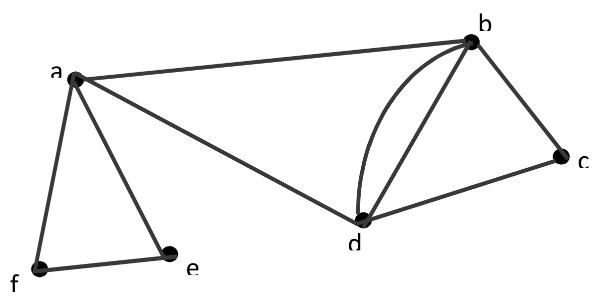

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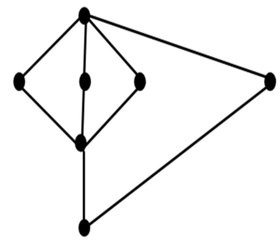

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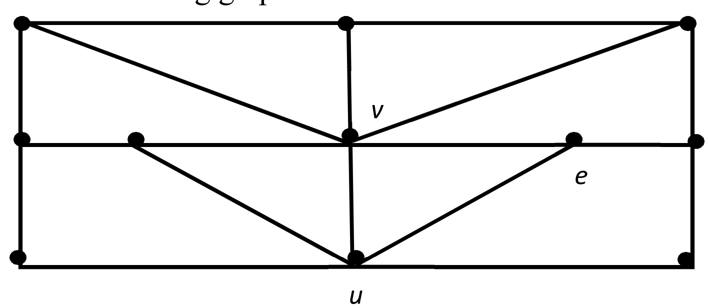

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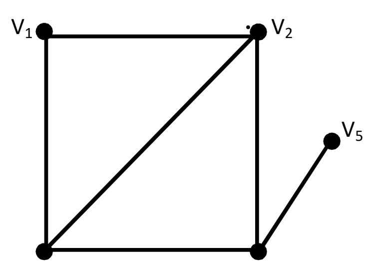

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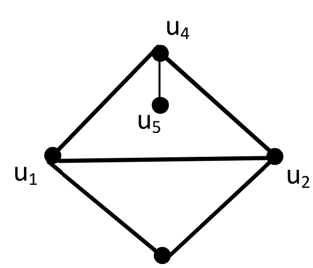

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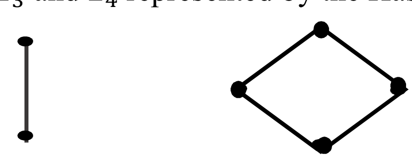

L_3 $\qquad\qquad$ L_4

Draw the Hasse diagram of lattice $L_3 \times L_4$.

Give example of a subset S of a lattice L, which is not a sublattice of L but is itself is a lattice.

Name of Course : **CBCS B.Sc. (H) Mathematics**

Unique Paper Code : **32357504**

Name of Paper : **DSE-II Mathematical Finance**

Semester : **V**

Duration : **3 hours**

Maximum Marks : **75 Marks**

Attempt any four questions. All questions carry equal marks.

1. Determine the effective rates of interest corresponding to the 5% rate of interest compounded daily, weekly, monthly, quarterly, semi-annually, and continuously. Arrange them in increasing order.

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Name of Course	: **CBCS B.Sc. (H) Mathematics**
Unique Paper Code	: **32357503**
Name of Paper	: **DSE- I, C++ Programming**
Semester	: **V**
Duration	: **3 hours**
Maximum Marks	: **75 Marks**

Attempt any four questions. All questions carry equal marks.

1. Write a function, *printgrid*, that takes two parameters, (one parameter is a two-dimensional array and the second parameter is for the number of rows) and print the following grid structure using repetition and controlled statements. After that call the function, *printgrid*, in the main program.

```
#
  #  #
  #  1#
  #  1  2  #
  #  1  2  3  #
  #  1  2  3  4  #
  #  1  2  3  4  5  #
  #  1  2  3  4  5  6  #
  #  1  2  3  4  5  6  7  #
  #  #  #  #  #  #  #  #  #  #
```

2. Write a function in C++ using the one dimensional array to calculate the following quantity:

$$\sqrt{\frac{\left|\sum_{i=1}^{n}(x_i - \bar{x})^3\right|}{n(n-1)}}$$

where,
x_i denotes the data stored in the cells of array
$\bar{x}$ denotes the average of the data stored in the array
n denotes the number of data stored in the array and $n > 1$

3. Write a program to find the inverse of a 3×3 matrix over the field Z_{23} entered by the user, if the determinant of the matrix is non-zero. The program also finds the sum of the square of the diagonal elements under modulo 23 of the inverse of the given matrix.

4. Write a program which finds the solution of the following system of equations by matrix inversion method.

$$2x + y + 2z = 0,$$
$$2x - y + z = 10,$$
$$x + 3y - z = 5.$$

5. Find the all errors of given program

```
        #include<iostream>;
        Using Namespace Std :
void Swap(int *x, int *y):
Intmain()
        {
    Int a, b, d;
        a = 4;
        b = 2;
    c = c + a;
    j = 1:
    for(k = 1; k<=n, k++)
            {
cout<<setw(4)<< j;
        j = j + 3
    d = d +Pow(k, 2);
    cout<<"the Value of l in the" <<k<<"th iteration is  "<<l<<Endl;
            }
    Double p;
    p = Sqrt{c, 3};
    cout<<p<<endl;
    Cout<<"Square root of "<< c<< "is  "<<p<<Endl;
int x = 5; y = 10;
cout<< "Before swap, x: " << x << " y: " << y <<Endl;
swap(&x, &y);
Cout<< "Main. After swap, x: " << x << " y: " << y <<Endl;
        Return 0:
        }
    void swap (int *px, int *py)
        {
    int temp;
    cout<< "Swap. Before swap, *px: " << *px<<" *py: " << *py<<endl;
    temp = *pX
        *pX = *pY
        *pY = temp
    cout<< "Swap. After swap, *px: " << *px<<" *py: " << *py<<endl;
        }
```

Write the correct program of part (a)

Write the equivalent programs by using **while** and **do... while** loops.

6. There are 20 students in a class. Write an appropriate code in C++ using arrays to generate the Internal Assessment Marks of a particular paper based on the following information:

i. Maximum Marks of the paper=100

ii. Roll Number, Student Name, Test Marks, Assignment Marks and Attendance Percentage are stored in arrays.

iii. Maximum Marks for Test = 10% of Maximum Marks of the paper
Maximum Marks for Assignment = 10% of Maximum Marks of the paper
Maximum Marks for Attendance = 5% of Maximum Marks of the paper

Attendance Marks based on Attendance percentage is given in following table:

67 % ≤ Attendance < 70 %	1 % of Maximum Marks for Attendance
70 % ≤ Attendance < 75 %	2 % of Maximum Marks for Attendance
75 % ≤ Attendance < 80 %	3 % of Maximum Marks for Attendance
80 % ≤ Attendance < 85 %	4 % of Maximum Marks for Attendance
85 ≤ Attendance	5 % of Maximum Marks for Attendance

iv. Internal Assessment Marks = Test Marks + Assignment Marks + Attendance Marks
Print the following details for each student-

Roll Number	Student Name	Test Marks	Assignment Marks	Attendance Marks	Internal Assessment Marks

Name of Course	: **CBCS B.Sc. (H) Mathematics**
Unique Paper Code	: **32357501**
Name of Paper	: **DSE-I NUMERICAL METHODS**
Semester	: **V**
Duration	: **3 hours**
Maximum Marks	: **75 Marks**

Attempt any four questions. All questions carry equal marks.

1. Consider the equation $2x - \log_{10} x - 7 = 0$ on $]3.78, 3.79[$. Find n, the number of iterations of bisection required to have an approximate root with absolute error less than or equal to 10^{-7}.

Find an approximate root for the equation $f(x) = x\sin x - 1 = 0$ using Regula-Falsi method. Do 2 iterations.

Use secant method to find an approximate root of the equation $f(x) = x^2 - 2x + 1 = 0$ starting with $x_0 = 2.6$ and $x_1 = 2.5$. Do two iterations. Compare the results with the exact root $1+\sqrt{2}$.

2. Find the maximum value of the step size h that can be used in the tabulation of $f(x) = \sin 2x$ on $[1,2]$ so that the error in the linear interpolation of $f(x)$ is less than 5×10^{-4}.

The function $f(x) = x^3 + 2x^2 - 3x - 1$ has a zero on the interval $[-3,-2]$ and another on the interval $[-1, 0]$. Approximate the largest negative zero using bisection method. Do 2 iterations.

The equation $x^2 + ax + b = 0$ has two real roots α and β. Consider a rearrangement of this equation as $x = -\dfrac{(ax+b)}{x}$. Show that the iteration $x_{i+1} = -\dfrac{(ax_i+b)}{x_i}$ will convergent be near $x = \alpha$, when $|\alpha| > |\beta|$.

3. Find an LU decomposition of the matrix $A = \begin{bmatrix} 1 & 4 & 3 \\ 2 & 7 & 9 \\ 3 & 8 & 2 \end{bmatrix}$ and use it to solve the system $AX = [1, 2, 8]^T$.

For Jacobi method, calculate T_{jac} and c_{jac} and the spectral radius of the coefficient matrix of the following system

$$\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} X = \begin{bmatrix} 1 \\ 3 \end{bmatrix}.$$

4. Solve the following system of equations using Gauss Seidel iteration method:

$$2x_1 - x_2 + x_3 = -1$$
$$-x_1 + 4x_2 + 2x_3 = 3$$
$$x_1 + 2x_2 + 6x_3 = 5$$

Take $X^{(0)} = [0, 0, 0]^T$ and iterate three times.

Solve the following system of equations using SOR iteration method:

$$2x_1 - x_2 + x_3 = -1$$
$$-x_1 + 4x_2 + 2x_3 = 3$$
$$x_2 + 6x_3 = 5$$

Take $w = 0.9$ with $X^{(0)} = [0, 0, 0]^T$ and iterate three times.

5. Approximate the derivative of $f(x) = \sin 2x$ at $x_0 = \pi/2$, taking $h = 1,\ 0.1,$ and 0.01 using the formula

$$f'(x_0) \approx \frac{f(x_0) - f(x_0 - h)}{h}$$

Find the order of approximation.

Construct the divided difference table for the following data set and then write out the Newton form of the interpolating polynomial. Hence, estimate the value of y when $x = 2$

x	-1	0	1	3
y	-12	-3	4	36

6. Approximate the second order derivative of $f(x) = e^{-x}$ at $x_0 = 0$, taking $h = 1, 0.1,$ and 0.01 by using the formula

$$f''(x_0) \approx \frac{f(x_0 - h) - 2f(x_0) + f(x_0 + h)}{h^2}$$

Find the order of approximation.

Approximate the solution of the initial value problem in 5 steps using Euler's method.

$$\frac{dy}{dx} = \frac{e^x}{y}, y(0) = 1, 0 \le x \le 2.$$

Also find the absolute error at each step given that the exact solution is $y(x) = \sqrt{2e^x - 1}$.

Name of Course	: **CBCS B.Sc. (H) Mathematics**
Unique Paper Code	: **32351502**
Name of Paper	: **C 12-Group Theory-II**
Semester	: **V**
Duration	: **3 hours**
Maximum Marks	: **75 Marks**

Attempt any four questions. All questions carry equal marks.

1. If Aut (G) and Inn (G) denote the set of all automorphisms and the set of all inner automorphisms, respectively of a group G, then obtain Aut $(\mathbb{Z}_8)$ and Inn (D_8), where D_8 is a dihedral group of order 8. Is Inn (D_8) isomorphic to $\mathbb{Z}_4$? Justify.

2. Express $U(40)$ as an External Direct Product and an Internal Direct Product of its subgroups. What are the possible order of elements in $U(40)$? How many elements are of order 4 in $U(40)$? Further, find the number of cyclic subgroups of order 4 in $U(40)$.

3. Determine the isomorphism classes of Abelian groups of order 500. Further find those isomorphism classes that have 'only one subgroup of order 5' and those isomorphism classes that has 'only six subgroups of order 5'.

4. Let $G = D_8$ be the dihedral group of order 8 with the usual generators r and s and $A = \langle sr \rangle$ be the subgroup in G. Find the centralizer of A, $C_G(A)$ and the normalizer of A, $N_G(A)$. Further, list the left costs of H, and label them with the integers 1,2,3,4. Exhibit the image of each element of G under the representation π_H of G into S_4 obtained from the action of G by left multiplication on the set of left cosets of H in G. Is the representation faithful? Justify.

5. Let $\sigma_1 = (25)(136)$ and $\sigma_2 = \sigma_1(65)$ be the permutations in S_7. Are σ_1 and σ_2 conjugate? If they are, give an explicit permutation τ such that $\tau\sigma_1\tau^{-1} = \sigma_2$. Also, discuss again for $\sigma_2 = \sigma_1^5$. Further find $|C_{S_7}(\sigma)|$, where $\sigma = (25)(136)$.

6. Let G be a group of order 735. If the number of Sylow 7-subgroups are more than 1, then show that there exists a normal Sylow 5-subgroup, K. Also show that there exists a normal subgroup of order 245. Further show that $K \subseteq Z(G)$.

Name of the course : **CBCS B.Sc. (H)Mathematics**

Unique Paper Code : **32357502**

Name of Paper : **DSE-I: Mathematical modelling and graph theory**

Semester : **V**

Duration : **3 hours**

Maximum Marks : **75 Marks**

Attempt any four questions. All questions carry equal marks.

1. Use the Laplace transform to solve the initial value problem $y'' + 3y' + 2y = e^{-t}$, $y(0) = y'(0) = 0$. Find $\mathscr{L}^{-1}\left(\frac{3s+6}{s^2+3s}\right)$, $\mathscr{L}^{-1}\left(\frac{1}{(s+3)^2+4}\right)$. Is the unit step function of exponential order? Justify your claim.

2. Show that $x = 0$ is an ordinary point for the differential equation $y'' - xy' - py = 0$ where p is any constant. Also find the power series solution of the above equation in powers of x. Find the radius of convergence of the following two series:

$\left(\frac{x}{2} + \frac{1.3}{2.5}x^2 + \frac{1.3.5}{2.5.8}x^2 + \ldots\right)$ and $\Sigma\frac{x^n}{n!}$.

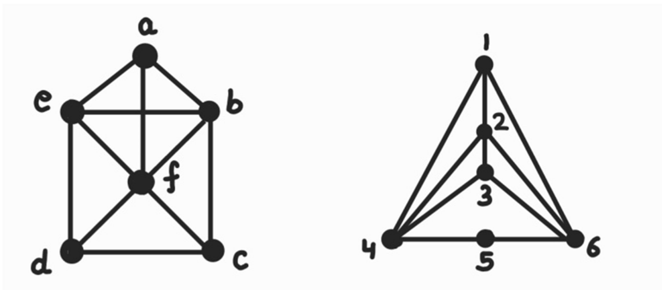
3. Using Monte Carlo simulation, write an algorithm to calculate the volume above xy-plane and below the paraboloid given by $z = x^2 + 2y^2$ for $(x, y) \in [0,1] \times [0,1]$. Use Linear Congruence method to generate 15 random real numbers with multiplier 2, increment 5, modulus 13 and seed 1. Is there cycling? If yes, then give the period of cycling.

Given below is the data for a hypothetical situation with four ships on a small harbor, using that answer the following : Average time of a ship in the harbor, maximum time of a ship in the harbor, average waiting time of a ship, maximum waiting time of a ship and percentage of time dock facilities are idle.

	Ship 1	Ship 2	Ship 3	Ship 4
Time between successive ships	30	35	40	60
Unload time	25	60	35	70

4. Formulate and solve the following linear programming problem using Simplex method. Also verify its solution using Algebraic or Geometric method: A container manufacturer is considering the purchase of two different types of cardboard-folding machines: model A and model B. Model A can fold 10 boxes per minute and requires 6 attendants, whereas model B can fold 20 boxes per minute and requires 8 attendants. Suppose the manufacturer can fold a maximum of 140 boxes per minute and cannot afford more than 72 employees for the folding operation. If a model *A* machine saves Rs. 8 and a model *B* machine saves Rs.12 per minute, how many machines of each type should be bought to maximize the savings?

5. Are the following graphs isomorphic? If so, find a suitable one-one correspondence between the vertices of graphs: if not, explain why no such correspondence exists. Are these graphs Hamiltonian. If yes, then write down a Hamiltonian cycle otherwise explain with reason. Check whether the conditions of Ore's theorem hold for both the graphs.



Draw two graphs each with 7 vertices and 9 edges: one that is Eulerian but not Hamiltonian and one that is Hamiltonian but not Eulerian.

6. Draw a simple graph with degree sequences 6, 4,4, 3, 3, 2, 1, 1 and find the total number of edges. How many continuous pen-strokes are needed to draw this simple graph without covering any part twice? Justify your answer. Can a simple graph have 5 vertices and 12 edges? If so, draw it, if not, explain why it is not possible to have such a graph. What is thelargest number of vertices in a graph with 35 edges if all vertices are of degree at least 3? There are 15 telephones in a town. Can we connect them so that each telephone is connected with the other 5 telephones?

Unique Paper Code	:	32377908
Name of the Paper	:	Econometrics (DSE – 2(B))
Name of the Course	:	B.Sc. (Hons.) Statistics
Semester	:	V
Duration	:	3 hours
Max Marks	:	75

Instructions for Candidates

1. Answer any **FOUR** questions.
2. All Questions carry **equal marks**.
3. The question paper has total 4 pages including this page

1. Suppose in the regression model:

$$Y_t = \alpha + \beta_1 X_{1t} + \beta_2 X_{2t} + U_t,$$

γ_{12} which depicts coefficient of correlation between X_1 and X_2 is zero.

   (a) Would you suggest to run the following regressions:

$$Y_t = \alpha_0 + \alpha_1 X_{1t} + U_{1t}$$

$$Y_t = \gamma_0 + \gamma_2 X_{2t} + U_{2t}$$

   Justify your answer.

   (b) Also examine whether:

   (i) $\hat{\alpha}_1 = \hat{\beta}_1$ and $\hat{\gamma}_2 = \hat{\beta}_2$

   (ii) $\hat{\alpha} = \hat{\alpha}_0$ and $\hat{\alpha} = \hat{\gamma}_0$

   Support your answer with appropriate expressions and their mathematical proof.

2. Suppose that a researcher , using data on class size (CS) and average test scores from 50 third-grade classes, estimates the OLS regression:

$$\widehat{TestScore} = 640.3 - 4.93 \text{ x } CS$$

with $R^2 = 0.11$, and estimate of error variance = 75.69

Last year a classroom had 21 students, and this year it has 24 students. What is the regression's prediction for the change in the classroom average test score?

Further, the sample average class size across the 50 classrooms is 22.8. What is the sample average of the test scores across the 50 classrooms?

Also find the sample standard deviation of test scores across the 50 classrooms.

3. Statement: "Absence of high pair-wise correlation(s) among regressors confirms absence of multicollinearity"

Is this statement true for a linear regression model with two independent variables? Give detailed reasons to support your answer.

Discuss the validity of the statement when we have more than two regressors in the model? In this case, if you think that the information provided by pair-wise correlations is insufficient, what kind of additional information will be required to conclude about the presence or absence of multicollinearity?

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4. A study was performed to assess the impact of employment size (X) on the average compensation (Y) and following regression was obtained

$$\hat{Y}_i = 3417.833 + 148.767\, X_i \qquad (*)$$

(81.136) (14.418)

p-value = (<0.001) (<0.001) $R^2 = 0.9383$

The analyst suspected violation in one of the assumptions and hence fitted the model

$$\frac{\hat{Y}_i}{\sigma_i} = 3406.639\frac{1}{\sigma_i} + 154.153\,\frac{X_i}{\sigma_i} \qquad (**)$$

(80.983) (16.959)

p-value = (<0.001) (<0.001) $R^2 = 0.9993$

What violation is the analyst suspecting? Identify the specific form of this violation for which regression (**) provides a solution. Explain any one test to detect this violation. What are the repercussions of such a violation?

5. For the General Linear model $\underset{nx1}{Y} = \underset{nx3}{X}\,\underset{3x1}{\beta} + \underset{nx1}{U}$ with usual assumptions, obtain the restricted least square estimator of $\underset{\sim}{\beta}$ along with its variance, under the set of linear restrictions given by $\begin{bmatrix} 2 & 1 & -1 \\ 1 & 0 & 1 \end{bmatrix}\underset{\sim}{\beta} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$.

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6. In an Election research, data were obtained for 72 districts of California State legislature and included the total number of registered voters by district, their party affiliation, the number of votes received by each candidate and the identity of the incumbent. The following are the results of three regression runs (numbers in parenthesis are *t* - values)

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All districts:

$$\text{WSV} = 0.240 + \underset{(4.82)}{0.174}\,\text{WSTE} + \underset{(4.60)}{0.141}\,\text{WSRV} + \underset{(7.01)}{0.075}\,\text{I} \quad ; \quad R^2 = 0.535,\ n = 72$$

Incumbent districts:

$$\text{WSV} = 0.329 + \underset{(3.67)}{0.157}\,\text{WSTE} + \underset{(6.07)}{0.409}\,\text{WSRV} \quad ; \quad R^2 = 0.440,\ n = 55$$

Nonincumbent districts:

$$\text{WSV} = 0.212 + \underset{(3.39)}{0.234}\,\text{WSTE} + \underset{(3.21)}{0.399}\,\text{WSRV} \quad ; \quad R^2 = 0.515,\ n = 17$$

where WSV = Winners share of total votes cast, WSTE = Winners share of total advertising expenditures, WSRV = Proportion of registered voters that are registered to the winner's political party and $I = \begin{cases} 1 & \textit{for incumbent district} \\ 0 & \textit{for Nonincumbent district} \end{cases}$

(a) Interpret the regression coefficients for *all districts*. Why is the coefficient of WSTE different in the three equations?

(b) ~~Explain exactly what the t-value means~~What do you infer from the t-value~~.~~? Interpret R^2 and enunciate on why it is different for each equation~~?~~.

(c) Why does the Incumbency variable appear only in the first equation?

(d) Could this model be used productively to predict? What insights could a candidate get from the model?

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Unique Paper Code : 32377905

Name of the Paper : Time Series Analysis (DSE-1(i))

Name of the Course : B.Sc. (Hons.) Statistics under CBCS

Semester : V

Duration : 3 hours

Maximum Marks : 75 Marks

Instruction for Candidates

Attempt any *four* questions. All questions carry equal marks. Use of a calculator is allowed.

1. State the conditions under which the Moving Average method can be recommended for trend analysis? How will you determine the period of the moving average? Calculate the 4-yearly moving average of the following data relating to sales in a departmental store:

Year	2000	2001	2002	2003	2004	2005	2006	2007
Sales (in crores of Rs.)	960	976	974	996	1024	1040	1688	1128

Year	2008	2009	2010	2011	2012	2013	2014	2015
Sales (in crores of Rs.)	1144	1120	1140	1168	1196	1212	1200	1180

Further show that centered moving average values of period 4 are same as the weighted moving average value of period 5 with weights 1,2,2,2,1.

2. Explain what is meant by seasonal fluctuations of a time series. A company manufactures bicycles. Given the quarterly production figures of the company for the last 4 years, explain

the procedure to compute seasonal indices by the 'link relatives' method. Use link- relatives method to compute seasonal indices from the recorded production figures given below:

YEAR	Q1	Q2	Q3	Q4
2016	420	414	502	365
2017	491	456	516	337
2018	463	365	478	310
2019	502	487	536	404

3. Describe the method for the estimation of the variance of the random component of a time series. For large samples, how will you check for the homogeneity of two successive estimates of variances? If ε_t is a random series, show that the correlation between successive items of $\Delta^k \varepsilon_t$ for long series is $\frac{-k}{k+1}$ and hence tends to -1 as k increases.

4. Explain what is meant by a weakly (or second-order) stationary process and define the autocorrelation function, ρ_u for such a process. Show that the ac.f of the stationary second order AR process

$$X_t = \frac{1}{12}X_{t-1} + \frac{1}{12}X_{t-2} + Z_t$$

is given by

$$\rho_k = \frac{45}{77}\left(\frac{1}{3}\right)^{|k|} + \frac{32}{77}\left(-\frac{1}{4}\right)^{|k|} ; k = 0, \pm 1, \pm 2, \ldots$$

Under what conditions is a second order MA process stationary?

5. Explain clearly the steps involved in Box-Jenkins approach to forecasting. For the model $(1 - B)(1 - 0.2B)y_t = (1 - 0.5B)\varepsilon_t$ classify the model as an $ARIMA(p, d, q)$ process. Determine whether the process is stationary and invertible. Evaluate the first three ψ weights of the model when expressed as an $MA(\infty)$ model. For the given ARIMA model, find the forecasts for one- and two-steps-ahead and show that a recursive expression for forecasts three or more steps ahead is given by

$$\hat{y}_n(h) = 1.2\hat{y}_n(h-1) - 0.2\hat{y}_n(h-2)$$

6. When is Simple Exponential Smoothing procedure an optimal method to use? What values can the smoothing constant take on? What is the impact of various

values of the smoothing constant on the smoothed time series? The table below shows the temperature (degrees C), at 11 p.m., over the last ten days:

Day	1	2	3	4	5	6	7	8	9	10
Temperature	1.5	2.3	3.7	3.0	1.4	-1.3	-2.4	-3.7	-0.5	1.3

Calculate a three-day moving average forecast for the temperature at 11 p.m. on day 11? Apply exponential smoothing with a smoothing constant of 0.8 to derive a forecast for the temperature at 11 p.m. Which of the two forecasts for the temperature at 11 p.m. on day 11 do you prefer and why?

Unique Paper Code: 32371502

Name of Paper: Statistical Computing using C/C++ Programming

Name of Course: B.Sc. (H) Statistics (CBCS)

Semester: V

Duration: 3 Hours

Maximum Marks: 75 Marks

Instruction for Candidates:

(i) All questions carry equal marks. Attempt any FOUR questions.

(ii) 5 marks are reserved for explaining the steps involved in obtaining the output.

1.

(i) What do you mean by preprocessor directive in C? Explain any two preprocessor directive? Also explain why do we recommend the use of parentheses for formal arguments used in macro definition? Give an example.

(ii) Develop functions to carry out the matrix multiplication and addition of two matrices A of order $\mathrm{m} \times \mathrm{m}$ and B of order $\mathrm{m} \times \mathrm{m}$ $(m \leq 5)$. Hence, write a C program to compute Y=(A+B)*(A+B). Print the results in file.

2.

(i) What is a pointer variable? How ~~it~~ is it declared? How is a value ~~is~~ accessed by pointer? What are the arithmetic operators that are permitted on pointers? How ~~they~~ are they useful in dynamic memory allocation? Explain two functions of dynamic memory allocation.

(ii) Write a C recursion function to find the value of $n!$. Using this function write a C program to compute $\binom{n}{r}$ for n= 8 and r = 0,1,2…n. Print the results in file.

3.

(i) What is a structure? How is a structure declared and how are its members ~~are~~ accessed? Can you define an array inside a structure? Define a structure called *tag* containing the following three members:

(a) A 28 element character array called *design*

(b) An integer quantity called *get*

(c) A double quantity called *know*

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(ii) Develop a function to sort an array of n numbers into ascending order. Hence, write a C-program using pointers to generate 25 random numbers from gamma distribution with parameters k and theta. Also compute the sample median using the function developed above.

4.

(i) Explain the output of the following program:

```
#include <stdio.h>
int fun (int*, int, int*);
int main (void)
{
        int a = 3, b=10;
```

```
        int c[5] = {9,14,3,15,6};
        a=fun(&a,b,c);
        printf("2. %d %d %d %d %d %d %d\n", a, b,
        c[0],c[1],c[2],c[3],c[4]);
        return 0;
}

int fun (int* px, int y, int* pz)
{
        int a=15;
        int* p;
        printf("1. %d %d %d\n",*px,y,*pz);
        for (p=pz; p<pz+5; ++p)
             *p=a+*p;
        return(*px + *pz + y);
}
```

(ii) Develop a C function to find the correlation coefficient for the given discrete data in the form { (x_i, y_i) i = 1, 2, 3, ……n ≤25}. Write a C program to fit a line Y = a + bX using the function developed above. Also compute the fitted Y_i's. Print the results in file.

5.

(i) Explain the output of the following program:

```
#include<stdio.h>
void main( )
{
    int a[20],i,ele,j;
    ele= 145;
    i=0;
    while(ele > 0)
    {
            a[i]=ele%2;
            i++;
            ele=ele/2;
    }
    for(j=i-1;j>=0;j++)
            printf("%1d",a[j]);
}
```

(ii) Given two independent samples (x_i, i=1,2….n_1) and (y_i, i=1,2…n_2) drawn from the Normal populations N(μ_1 , σ^2) and N(μ_2 , σ^2) respectively, write a C-program to test for the equality of two means using t test. (Use of dynamic memory allocation is must.)

6.

(i) Given the following definitions:

int num[26] = {23, 3, 5, 7, 4,-1, 6};
*int * n = num, i=2, j=4;*
Evaluate of the following:

(a) n
(b) *n
(c) *n+1
(d) *(n+1)
(e) *n + j
(f) *&i
(g) *(n + i) +j
(h) *(n+i+j)
(i) *(num + i) + * (num+j)
(j) *(num + *(num+1))

(ii) Write a C-code to generate the 100 random numbers following ~~N(μ=20,σ=2)~~ N(20, 4) using central limit theorem. Calculate the sample mean and variance and compare with the statistics based on the population parameters. Read the parameters from the user and print the result in a file.

Unique Paper Code	**: 32351501**
Name of Course	**: B.Sc. (H) Mathematics**
Name of Paper	**: C11-Metric Spaces**
Semester	**: V**
Duration	**: 3 Hours**
Maximum Marks	**: 75**

Attempt any four questions. All questions carry equal marks.

1. Prove that a metric space (X, d) is compact if and only if every collection of closed sets in X with empty intersection has a finite subcollection with empty intersection.

Do there exist onto continuous functions

$$f_1: [0,1] \to \mathbb{Q}$$
$$f_2: [0,1] \to [2,3] \cup [4,5]$$
$$f_3: [0,1] \to (5,7)$$
$$f_4: [0,1] \to [2,4]$$
$$f_5: [0,1] \to (10, +\infty)$$
$$f_6: [0,1] \to (0,1]$$
$$f_7: (1, \infty) \to (0,1).$$

If yes, give the explicit expression for the function. If no, then clearly state the reason. **(4.75+14)**

2. Let $X = \mathbb{R}$, d_1 be the usual metric and d_2 be the discrete metric. Let $A = (0,1) \cup \{5\}$. Find the interior, derived set, closure and diameter of A in the metric spaces (X, d_1) and (X, d_2). Also, find the distance between the point 7 and the set A in the metric spaces (X, d_1) and (X, d_2).

Give an example of an open dense subset of (X, d_1) which is uncountable. Are the metric spaces (X, d_1) and (X, d_2) separable? Justify.

Prove that the function $f: (X, d_2) \to (X, d_1)$ defined by $f(x) = x$ is continuous, one-one and onto, but not a homeomorphism. **(8+2+1+4+3.75)**

3. Let $X = \mathbb{R}^2$ with Euclidean metric d. Prove that (X, d) is a complete metric space.

Verify the Cantor Intersection Theorem for the sequence (F_n) of subsets of X, where $F_n = \bar{S}((0,0), 1/n)$ where $\bar{S}(x, r)$ denotes the closed ball centred at x and radius r.

Determine which of the following subsets of $(\mathbb{R}^2, d)$ are complete? Justify your answer in each case.

$$A_1 = \{(x, y): y = x\}$$
$$A_2 = \{(x, y): x > 0\}$$
$$A_3 = \{(x, y): x = 0\} \cup \{(x, y): y = 0\}$$
$$A_4 = \{(x, y): 1 < y < 2\}.$$

Prove that the metrics d and d_∞ are equivalent on $\mathbb{R}^2$, where $d_\infty(x, y) = \max\{|x_1 - y_1|, |x_2 - y_2|\}$ for $x = (x_1, x_2)$ and $y = (y_1, y_2)$. Is (X, d_∞) a complete metric space? Justify. **(4+4+4+4+2.75)**

4. Let $X = \mathbb{R}$ with usual metric and $Y = [0,1)$. Show that the subsets $F_1 = [0,0.2]$ and $F_2 = [0.8,1)$ are closed in Y. Also, show that the subsets $G_1 = (0.1,0.2)$ and $[0,0.4)$ are open in Y. Find the interior and closure of the subsets $(0,0.5), (0.5,1)$ and $[0,1)$ in Y. Prove that Y is a separable metric space. Is Y complete? Is Y connected? Is Y compact? Justify your answer in each case. **(2+2+6+2.75+2+2+2)**

5. If (X, d_X) is a disconnected metric space, prove that there exists a continuous function $f: (X, d_X) \to (\mathbb{R}, d)$ which does not have Intermediate Value Property. Hence or otherwise prove that every continuous function f on the interval $[-1, 1]$ with $|f(x)| \leq 1 \; \forall x \in [-1, 1]$ has a fixed point.

What can be said about the connectedness of $\mathbb{Q}$, w.r.t. the metric

$$d(x, y) = \frac{|x-y|}{1+|x-y|}?$$

Justify.

For a subset Y of $\mathbb{R}$ such that $(0, 1) \subset Y \subset [0, 1]$, what can be concluded about its connectedness and why ?

Is the diameter of a connected set zero? Justify your answer by an example.

Give an example of an infinite connected set with finite diameter. **(5+4+4+2+1.75+2)**

6. For a metric space (X, d) and a continuous function f from X into itself, show that the set of points $\{x: f(x) = x\}$ is a closed subset of X.

If f is a continuous function from a compact metric space (X, d_X) into an arbitrary metric space (Y, d_Y), prove that f is uniformly continuous and the image of X under f is compact.

Further, in addition if (X, d_X) and (Y, d_Y) both are homeomorphic, what can be concluded about Y? Justify.

Hence prove that there exist $x_1, x_2 \in [a, b]$ such that $f(x_1) = \sup_{x \in [a,b]} f(x)$ and $f(x_2) = \inf_{x \in [a,b]} f(x)$, where $f: [a, b] \to \mathbb{R}$ is such that $f(t) = \alpha t + \beta$, for some α and β in $\mathbb{R}$.

(5+6+3+3.75)

Unique Paper Code:	32371501
Name of the Course:	B.SC (Hons.) STATISTICS: CBCS
Name of the Paper:	Stochastic Processes and Queuing Theory
Semester:	V
Duration:	3 hours
Maximum Marks:	75

Instructions for Candidates:

1. Attempt ***four*** questions in all.
2. All questions carry equal marks.
3. Use of calculator is allowed.

1. Let $\{X_n, n \geq 0\}$ be a Markov Chain having state space $S = \{0,1,2,3,4,5,6\}$ and transition matrix

$$\begin{pmatrix} 0 & 0 & \frac{3}{4} & \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 \\ \frac{1}{5} & 0 & \frac{2}{5} & \frac{2}{5} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}.$$

Find the nature of each state. Also, identify the closed set(s), if any and obtain its stationary distribution (if it exists).

2. Let X be a zero- one truncated Poisson variate with parameter (λ) with zero-one classes missing and having the p.m.f,

$$P(X = k) = \frac{e^{-\lambda}\lambda^k}{k!(1 - e^{-\lambda} - \lambda e^{-\lambda})}, k = 2, 3, ...$$

Obtain the probability generating function of r.v X and hence obtain mean and variance of X. Further, if X_1 and X_2 are i.i.d zero- one truncated Poisson variates with parameter (λ), find the probability generating function of $X_1 + X_2$.

3. Derive the differential difference equations for the linear growth process with immigration/emigration having

$\lambda_n = n\lambda + 4;\ \mu_n = n\mu - 3$

Hence obtain the second moment differential equation of mean population size when population starts with 3 individuals at t=0.

4. Under queuing model when arrival and departure follow Poisson process with single server , 5 as space limit for customers and FIFO queue discipline, derive the steady-state probabilities and obtain the mean size of the system . Further, modify above equations results when system has no space limit .

5. Construct the transition probability matrix when game starts with \$8 and gambler with initial capital \$3 starts the game and his probabilities of winning, losing and drawing a game are (0.46, 0.44,.10) respectively with stopping criteria is either gambler ruins or wins all amount. Further, find expected duration of the game when gambler either wins or loses a game with equal probability and no possibility of draw.

6. Assume that total number of COVID-19 cases follow Poisson process with parameter 6 per day. The COVID-19 patient recovery probability is 0.88 and probability of death is 0.12. Find the mean and variance of recovered cases at the end of 10^{th} day. Find, the probability that the number of recovered cases exceed number of deaths by 4 at the end of 2^{nd} day.