Unique paper Code	:	32371303
Name of the Paper	:	Mathematical Analysis
Name of the Course	:	B.Sc. (H) Statistics Under CBCS
Semester	:	III
Duration	:	3 Hours
Maximum Marks	:	75

Instructions for Candidates

1. **Attempt any four questions.**
2. **All Questions carries equal marks ~~18.75 marks each~~.**

1. (a) Define

(i) Neighbourhood of a point,
(ii) Open Set and
(iii) Closed Set.

Give an example of each of above.

Also show that the union of an arbitrary family of Closed sets may

(i) be a Closed Set
(j) fail to be a Closed Set

(b) Define Supremum and infimum of a Set. Find ~~these~~ supremum and infimum for the following set ~~S where~~

$$S = \{ x \in R : x^2 - 2x - 5 < 0 \}.$$

(c) Examine the following set for its limit point/~~(s)~~

$$S = \left\{ \frac{1}{n} , \; n \in Z^+ \right\}.$$

~~(6, 4, 8.75)~~

2. (a) Define a Convergent Sequence. Use the definition to show that $\lim_{n\to\infty} \frac{3+2\sqrt{n}}{\sqrt{n}} = 2$.

(b) Define Convergence and Absolute Convergence of a Series. Is every convergent series absolute convergent ? Justify your answer. Test for convergence of the series

$$1 - \frac{1}{2!} + \frac{1}{4!} - \frac{1}{6!} - \ldots$$

~~(7,11.75)~~

3. (a) Test for the convergence of the series

i) $\sum_{n=1}^{\infty} (1/n)^{1/n}$

ii) $\frac{\alpha}{\beta} + \frac{1+\alpha}{1+\beta} + \frac{(1+\alpha)(2+\alpha)}{(1+\beta)(2+\beta)} + \ldots.$

(b) Obtain Maclaurin's series expansion of $f(x) = (1+x)^m \quad \forall\, x \in R$, where m is a positive integer.

~~(5+7, 6.75)~~

4. (a) State and prove Rolle's Theorem. Give its ~~Geometrical~~ geometrical ~~Interpretation~~interpretation. Further prove that if **p** is a polynomial and **p**′ the derivative of **p,** then between any two consecutive zeroes of **p**′, there lies at most one zero of **p.**

(b) Let f be the function defined on R by setting

$$f(x) = \begin{cases} |x-1| + |x+2|, & x \neq 0 \\ 0 & x = 0 \end{cases}$$

Examine the function f for continuity and derivability at x = 1 and x = -2.

~~(10.75,8)~~

5. (a) Derive a formula which can be used for interpolating a value of f(x) near the end of the tabulated values.

(b) Identify the following expression and derive ~~the same~~it by defining the appropriate conditions~~.~~

$$f(x) = f(x_0) + (x - x_0)f(x_0, x_1) + \cdots + (x - x_0)(x - x_1) \ldots (x - x_n)f(x_0, x_1, \ldots\ldots, x_n)$$

(c) Use method of separation of symbols to prove the identities

$$u_0 + \binom{n}{1}u_1 x + \binom{n}{2}u_2 x^2 + \cdots + \binom{n}{n}u_n x^n = (1+x)^n u_0 + \binom{n}{1}(1+x)^{n-1}x\Delta u_0 + \binom{n}{2}(1+x)^{n-2}x^2\Delta^2 u_0 + \cdots \ldots \ldots + x^n\Delta^n u_0$$

~~(7,7,4.75)~~

6 (a) Define~~d~~ the operators E, Δ, μ, δ and ▽ and show that

$$\mu = \frac{1}{2}\left(E^{\frac{1}{2}} + E^{-\frac{1}{2}}\right) = \frac{2+\Delta}{2\sqrt{1+\Delta}} = \sqrt{1 + \frac{1}{4}\delta^2}$$

(b) Calculate approximations to the value of $\int_0^6 \frac{dx}{1+x^2}$ by using ~~simpson's~~ Simpson's one-third rule.

~~(9.75, 9)~~

Unique Paper Code : 32371301

Name Of The Paper : Sampling Distributions

Name of the Course : B.Sc.(H) Statistics(under CBCS)

Semester : III

Duration : 3 hrs

M.Marks : 75

Instructions for candidates:

1. All questions carry equal marks
2. Attempt any four questions.

1) If X is a random variable and $E(X^2) < \infty$, then prove that $P(|X| \geq a) \leq E(X^2)/a^2$, $\forall a \geq 0$. If $\{X_k\}$ is a sequence of independent random variables such that $P(X_k = \pm k^{\alpha}) = \frac{1}{2}$, examine whether WLLN and central limit theorem hold for the sequence $\{X_k\}$.

2) Suppose $X_1, X_2, \ldots\ldots, X_n$ $(n > 1)$ are independent variates each distributed as $N(0, \sigma^2)$.

Find the p.d.f. of $W = \dfrac{X_1}{\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}}$ Why does W not follow the t-distribution?

If $V = \dfrac{W\sqrt{\sum_{i=1}^{n} x_i^2}}{\sqrt{\sum_{i=2}^{n} x_i^2}} \times \sqrt{\dfrac{n-1}{n}}$, obtain the distribution of V.

3) Show that for F- distribution with (n_1, n_2) d.f. , mean is greater than mode. Also discuss the shape of probability curve of F-distribution.

4) Let $X_1, X_2, \ldots, X_n$ be a random sample of size n with common p.d.f.

$$f(x) = \begin{cases} 1, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

Find the mean and variance of r^{th} order statistic $X_{(r)}$. Also find COV $(X_{(1)}, X_{(n)})$.

5) Let $X_1, X_2, \ldots, X_n$ be a random sample from $N(\mu, \sigma^2)$. Derive the p.d.f. of S^2, where $S^2 = \dfrac{1}{n-1}\sum_{i=1}^{n} (x_i - \bar{x})^2$. Also find $E(S^{2k})$. Hence or otherwise find $E(S^2)$and $Var(S^2)$.

6) The mean yield of two sets of plots and their variability are as given below Examine.

(i) Whether the difference in the mean yields of two sets of plots is significant and

(ii) Whether the difference in the variability in yields is significant.

	Set of 40 PLots	**Set of 60 Plots**
Mean Yield per plot	1258kg	1243kg
S.D. per Plot	34kg	28kg

Also obtain the 95% confidence interval for the difference of means.

Unique Paper Code: 32373902

Name of the Paper: Statistical Data Analysis Using R (SEE-2)

Name of the Course: B.Sc. (Hons.) Statistics

Semester: III

Duration: 2 hours

Max Marks: 50

Not as per OBE instructions

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Instructions for candidates

Attempt any FOUR questions. Write R codes for each question given in Section B along with other question related answers.

Section A

Fill in the blanks:

Q1(a) R code used to append an observation to a vector L is given by _____. 1

(b) A command used to extract 4^{th} and 6^{th} element from a vector x of 8 elements is _____. 1

(c) In R missing values are represented by _____ which should be in capital letters. 1

(d) Graphical window can be divided into several parts using the graphical instruction _____. 1

(e) A command/R code abline (v = value) is used for drawing _____ line. 1

(f) Write a statement/command to install a package to be used in R. Also, loads the same package for the current session of R. $1\frac{1}{2}$

(g) Write the arguments used in graphical representation of R for the line type and line width. $1\frac{1}{2}$

(h) Write R codes to obtain $P(X \leq 4), where\ X \sim Binomial(n = 15, prob. = 0.6)$. $1\frac{1}{2}$

(i) Write the output of the following R Codes: $1\frac{1}{2}$

```
X <- seq(10,90,20)
X
```

(j) Can we use customized x –axis limits in a graphical representation? Give example. $1\frac{1}{2}$

Section B

Q2 Given the frequency distribution $x_i|f_i$, having equal class intervals, draw less than and more than ogives in a single plot and also find the median. Also draw another plot for a histogram. $12\frac{1}{2}$

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Q3 Write R-Code to $12\frac{1}{2}$

1) construct boxplot and qq plot to check the normality of the parent population.

2) if the parent population is normal then construct 95% and 90% confidence interval for the population mean.

Q4 Write a R- code for revealing the effect of increasing the values of parameter lambda in a Poisson distribution. $12\frac{1}{2}$

Q5 t-test for difference of means when the samples are drawn from same population. Draw your conclusions based on 10% level of significance. Also interpret the results as obtained in R. Write R codes for mean, variance, median and mode for both the samples used in the above t-test. $12\frac{1}{2}$

Q6 Write a R- code for the following $12\frac{1}{2}$

(i) Draw a SRSWOR of size 20 from the population data frame Y of 100 students. Data frame has 3 fields viz. Name, RollNo and Marks.

(ii) Calculate sample mean, variance and population mean and variance of a field Marks.

(iii) Which library is required to be installed for above function.

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