Name of Course	: **CBCS (LOCF) B.Sc. (H) Mathematics**
Unique Paper Code	: **32351101**
Name of Paper	: **BMATH 101- Calculus**
Semester	: **I**
Duration	: **3 hours**
Maximum Marks	: **75**

Attempt any four questions. All questions carry equal marks.

1. Sketch the graph of the function $f(x) = x^3 - 6x + \frac{5}{2}$ by finding intervals of increase and decrease, relative extrema, concavity and inflection points (if any).

Evaluate the limit: $\lim_{x \to -\infty} (1 - \frac{1}{2x})^{5x}$.

A shoe factory can produce shoes at a cost of \$70 a pair and estimates that if they are sold for x dollars a pair, customers will buy approximately $N(x)$ number of pairs per week, where $N(x) = 500\, e^{-0.2x}$. Find the price at which the shoe pairs be sold so as to maximize the gain.

2. Find all horizontal and vertical asymptotes to the curve $y = \frac{x^3+8}{x^3-8}$. Does this curve have any cusp or vertical tangent? Explain.

Find the n^{th} derivative of the function $y = 3\sin^3 x$.

Obtain a reduction formula for $\int \operatorname{cosec}^n x\, dx$ and hence evaluate $\int \operatorname{cosec}^7 x\, dx$.

3. Sketch the graph of the curve $r = 1 - 2\sin\theta$ in polar coordinates.

Describe the graph of the equation: $5x^2 + 9y^2 - 20\,x + 54y + 56 = 0$.

Identify and sketch the curve: $3x^2 + 2\sqrt{3}\,xy + y^2 - 8x + 8\sqrt{3}\,y = 0$.

4. Find the arc length of the parametric curve: $x = (1 + t^2),\ y = (1 + t^3)$ for $0 \le t \le 1$.

Find the area of the surface generated by revolving the curve $y = \sqrt{25 - x^2}$, $-2 \le x \le 2$ about x-axis.

The region bounded by the curves $y = x$ and $y = x^2$ is rotated about the line $y = 4$. Compute the volume of the resulting solid.

5. Given $9\sinh x - \cosh x = 5$. Find the exact value of $\tanh x$.

Show that the vector- valued function given by

$$\boldsymbol{R}(t) = (2\hat{\boldsymbol{\imath}} + 2\hat{\boldsymbol{\jmath}} + \hat{\boldsymbol{k}}) + \left(\frac{1}{\sqrt{2}}\hat{\boldsymbol{\imath}} - \frac{1}{\sqrt{2}}\hat{\boldsymbol{\jmath}}\right)\cos t + \left(\frac{1}{\sqrt{3}}\hat{\boldsymbol{\imath}} + \frac{1}{\sqrt{3}}\hat{\boldsymbol{\jmath}} + \frac{1}{\sqrt{3}}\hat{\boldsymbol{k}}\right)\sin t$$

describes the motion of a particle moving in the circle of radius 1, centered at the point $(2, 2, 1)$ and lying in the plane $x + y - 2z = 2$.

A ball is thrown upward from the edge of cliff at an angle 30^o with initial speed 68 ft./s. Suppose the height of the cliff from the ground is 50 ft., then find the velocity and speed of the ball at the time of impact. Also, find the highest point where, the ball reached during the flight.

6. Find the integral $\int_1^e \frac{dx}{x\sqrt{1+(\ln x)^2}}$.

A shell is fired from the ground level with muzzle speed of 450 ft./s at an angle of 30°. An enemy gun 23,000 ft. away fires a shot 2 seconds later and the shells collided 45 ft. above the ground at the same speed. What are the muzzle speed V_o and angle of elevation α of the enemy gun?

Suppose $\boldsymbol{R}(t) = t\hat{\boldsymbol{\imath}} + 2t^2\hat{\boldsymbol{\jmath}} + t^3\widehat{\boldsymbol{k}},\ t \geq 0$ is the position vector of a moving object. Find the tangential and normal component of objects acceleration.

Unique Paper Code : **32351102**

Name of Paper : **BMATH102- Algebra**

Name of Course : **CBCS B.Sc.(H) Mathematics**

Semester : **I**

Duration : **3 hours**

Maximum Marks : **75 Marks**

Attempt any four questions. All questions carry equal marks.

1. Solve the following equations

(i) $x^4 - 6x^3 + 26x^2 - 96x + 160 = 0$, given that sum of two of the roots is 6.

(ii) $x^4 - 8x^3 + 31x^2 - 106x + 34 = 0$, given that $3 - \sqrt{7}$ is a root of the equation.

(iii) $x^3 - 24x^2 - 128x + 2048 = 0$, given that it has an integral root.

2. Find the polar representation of the complex number, $z = \frac{(1+i)^8\ (\sqrt{3}+i)^5}{(-1-i\sqrt{3})^8}$ and determine $|z|$, arg z, arg $\bar{z}$ and arg$(-z)$.

Solve the equation: $z^5 = 8(i\sqrt{3} - 1)$.

3. Define a relation $\sim$ on the set of integers $\mathbb{Z}$ by $u \sim v$ if and only if $5u + 3v$ is a multiple of 8. Prove that $\sim$ is an equivalence relation on $\mathbb{Z}$. Find the equivalence class of integers 0, 2 and 3. Determine the quotient set for this equivalence relation.

Consider a real function $f : \{u \in \mathbb{R}: u \neq -3/5\} \to \mathbb{R}$ defined by $f(u) = \frac{1}{5u+3}$. Show that f is one-one. Find the range of the function f and determine f^{-1}, if it exists.

4. Let S be an infinite set and let x, y be elements not in S. Prove that S and $S \cup \{x, y\}$ are the sets of same cardinality. Deduce that the intervals (2,3) and [2,3] have the same cardinality.

Evaluate 54^{25} (mod 11).

5. Consider the following system of linear equations:

$$x + 9y - z = 27$$
$$x - 8y + 16z = 10$$
$$2x + y + 15z = 37.$$

Write the matrix equation $Ax = b$ and the corresponding vector equation for the above system of linear equations. Find the general solution of the above system of equations by reducing the augmented matrix to row reduced echelon form. Deduce the general solution in parametric vector form for the homogeneous system $Ax = 0$. Find a basis and the dimension for the null space of A.

6. Find the characteristic polynomial, the eigen values and the corresponding eigen vectors and eigen spaces of the following matrix:

$$A = \begin{bmatrix} 5 & 2 & 0 \\ 2 & 5 & 0 \\ -3 & 4 & 6 \end{bmatrix}.$$

SET-A (New Course)

Unique paper code : 32371109

Name of the paper : Calculus

Name of the course : B.Sc.(Hons) Statistics (CBCS)

Semester : I

Duration : 3 Hours

Max. Marks : 75 Marks

Instructions for candidates

Attempt four questions in all. All questions carry equal marks.

1 i) Prove ~~Euler,s~~Euler's theorem for the function

$z = (x + y)\, \Psi(\frac{y}{x})$, where Ψ is any arbitrary function.

ii) Use L'Hopital's rule to evaluate the following:

a) $\lim_{x\to 0} \frac{xe^x - \log(1+x)}{x^2}$ (b) $\lim_{x\to 1}[\,(x-1)\tan(\frac{\pi x}{2})]$

2 i) Determine any local maxima or local minima of the function:

$f(x,y,z) = x^2 + y^2 + z^2$ subject to the constraint $x + 2y - 4z = 5$.

ii) If $l = x(1-r^2)^{-1/2}$, $m = y(1-r^2)^{-1/2}$, $n = z(1-r^2)^{-1/2}$, where $r^2 = x^2 + y^2 + z^2$,

then show that $J(l, m, n) = (1-r^2)^{-5/2}$.

3 i) Form partial differential equation by the elimination of the constants a and b from

$(x-h)^2 + (y-k)^2 + z^2 = c^2$.

ii) Use Charpit's method to find the complete integral of $z^2(p^2z^2 + q^2) = 1$.

4 i) Solve $D^2 - 2D + 4y = e^x \cos x$.

ii) If $y = \sin(m\sin^{-1}x)$, then show that

$(1-x^2)y_{n+2} = (2n+1)xy_{n+1} + (n^2 - m^2)y_n$ and find $y_n(0)$.

5 i) Find the sum of the series sum as $n\to\infty$,

$\frac{n+1}{n^2+1^2} + \frac{n+2}{n^2+2^2} + \frac{n+3}{n^2+3^2} + \cdots \frac{1}{n}$.

ii) Prove Duplication formula. Use it to show that

$\beta(m, n) * \beta(m+½, m+½) = \pi m^{-1} 2^{1-4m}$.

6 i) Evaluate the following double integral:

$$\int_0^a \int_0^{\sqrt{a^2-y^2}} \sqrt{(a^2 - x^2 - y^2)}\ dydx.$$

Change the order of integration in the integral:

$$\int_0^a \int_{\sqrt{ax-x^2}}^{\sqrt{ax}} V dxdy.$$

ii) Solve:

$x(z^2 - y^2)p + y(x^2 - z^2)q = z(y^2 - x^2)$, where p = $\frac{\partial z}{\partial x}$ and q = $\frac{\partial z}{\partial y}$.

Unique paper code	**:**	**32371101**
Name of the course	**:**	**B. Sc (H) Statistics under CBCS**
Name/Title of the paper	**:**	**Descriptive Statistics**
Semester	**:**	**I**
Duration	**:**	**3 hours**
Maximum marks	**:**	**75**

Instructions for candidates

Attempt any **four** questions. All questions carry equal marks.

~~Q~~1. Obtain the m.g.f of a random variable X having p.d.f

$$f(x) = \begin{cases} x & \text{for } 0 \le x \le 1 \\ 2 - x & \text{for } 1 \le x \le 2 \\ 0 & \text{elsewhere} \end{cases}$$

Find first four moments about origin. Hence find mean, standard deviation, coefficient of skewness and coefficient of kurtosis. Interpret your result also.

~~Q~~2. The probability density function of a random variable X is given by

$$f(x) = \frac{1}{\pi(1+x^2)} \ , -\infty \le x \le \infty.$$

If random variable X is transformed to a random variable Y by means of transformation $Y = \tan^{-1}(X)$ and further random variable Y by means of transformation $U = -2\log Y$. Find probability density function of random variable U.

~~Q~~3. Four identical tickets marked 1, 2, 3 and 123 respectively are put in an urn and one is drawn at random.
Let A_i denote the event that the number i appears on the drawn ticket, i =1, 2, 3.

(i) Are A_1 and A_2 independent?

(ii) Comment on the independence of events A_1, A_2 and A_3.

~~Q~~4. There are two bags A and B. A contains 4 white and 2 black balls and B contains 2 white and 4 black balls. One of the two bags is selected at random and two balls are drawn from it without replacement.

(i) Find the probability that the balls drawn are 1 black and 1 white.

(ii) Given that 1 black and 1 white ball~~s~~ are drawn, what is the probability that bag A was used to draw the balls?

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~~Q~~5. If X and Y are two random variables having joint p.d.f

$$f(x,y) = \begin{cases} 1/8\,(6-x-y) & ; \quad \text{for } 0 \le x \le 2 , 2 \le y \le 4 \\ 0 & ; \quad \text{elsewhere} \end{cases}$$

Find

(i) marginal p.d.f of X. Hence find mean of X.

(ii) conditional p.d.f of (Y | X= 1).

Also comment on independence of X and Y.

~~Q~~6. In a university examination, 100 children took three tests A, B and C. 40 passed the first, 39 passed the second and 48 passed the third test. 10 children passed all the three ~~tests~~tests, and 21 children failed all the three tests. 9 children passed the first two and failed the third and 19 failed the first two and passed the third.

(i) Find how many children passed at least two tests.

(ii) For the question asked, ~~certain~~ some of the given frequencies are not necessary. Which are they?

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