

Mathematics Teaching in the Online Mode: Pedagogical Opportunities and Challenges

An online certificate course for pre-service mathematics teachers

Organised by
Internal Quality Assurance Cell, LSR

A Report

Inaugural session

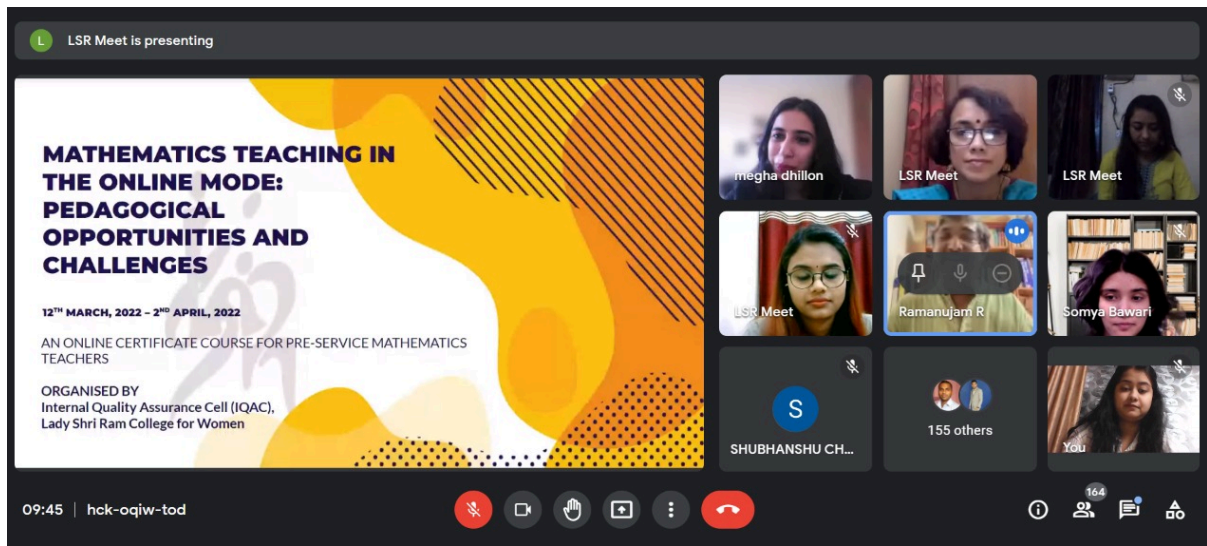
The inaugural session of the certificate course was held from 9:30 to 10:00 am on 12th March 2022. Prof. Suman Sharma, Principal, Lady Shri Ram College, delivered the welcome address and welcomed all the 160 participants from various colleges/departments of Delhi University. Her inspirational words helped set the context for the course. She also extended a warm welcome to Prof. R Ramanujam, the inaugural session speaker who was to conduct the first academic session of the course. This was followed by an address by Dr. Megha Dhillon, IQAC Convener, in which she elaborated on the different initiatives taken up by IQAC LSR, as a part of the academic outreach. Finally Dr. Jonaki B Ghosh, the course coordinator, introduced the theme of the course. She highlighted the objectives of the course and discussed the overall structure and modalities, which were to be followed by participants throughout the course. The logistics of the inaugural session and the 12 academic sessions of the course were managed by the 4th year Pedagogy of Mathematics students of the Department of Elementary Education under the guidance and supervision of Dr. Jonaki B Ghosh. The registered participants were from the following institutions.

Students from B.El.Ed course

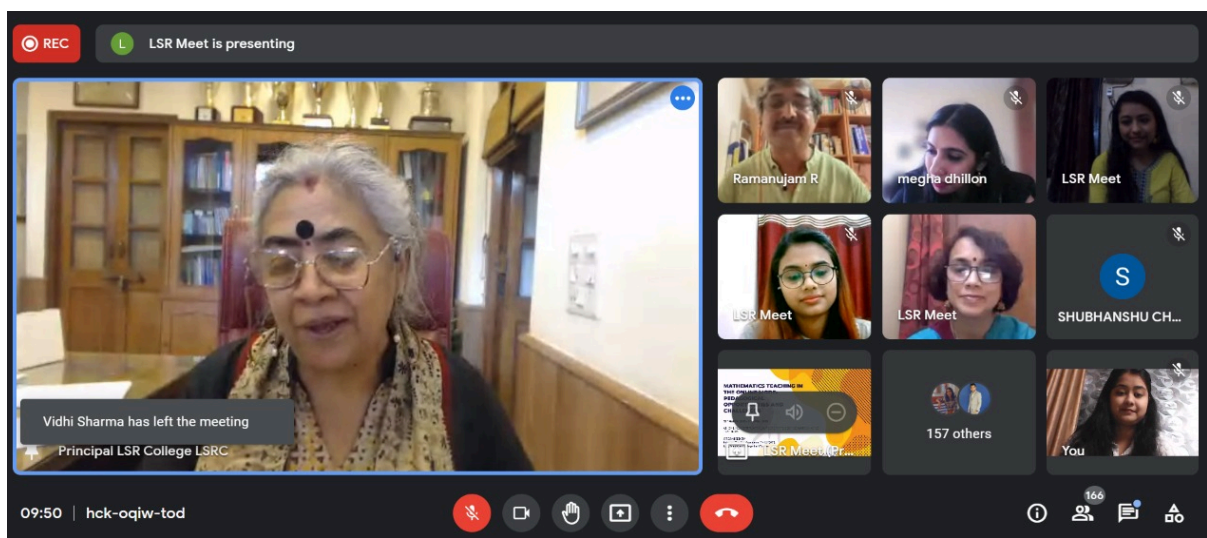
Aditi mahavidlaya
Miranda House
Shayama Prasad Mukherjee College
Mata Sundri College
Jesus and Mary College
Institute of Home Economics
Gargi College
Lady Shri Ram College

Students from B.Ed course

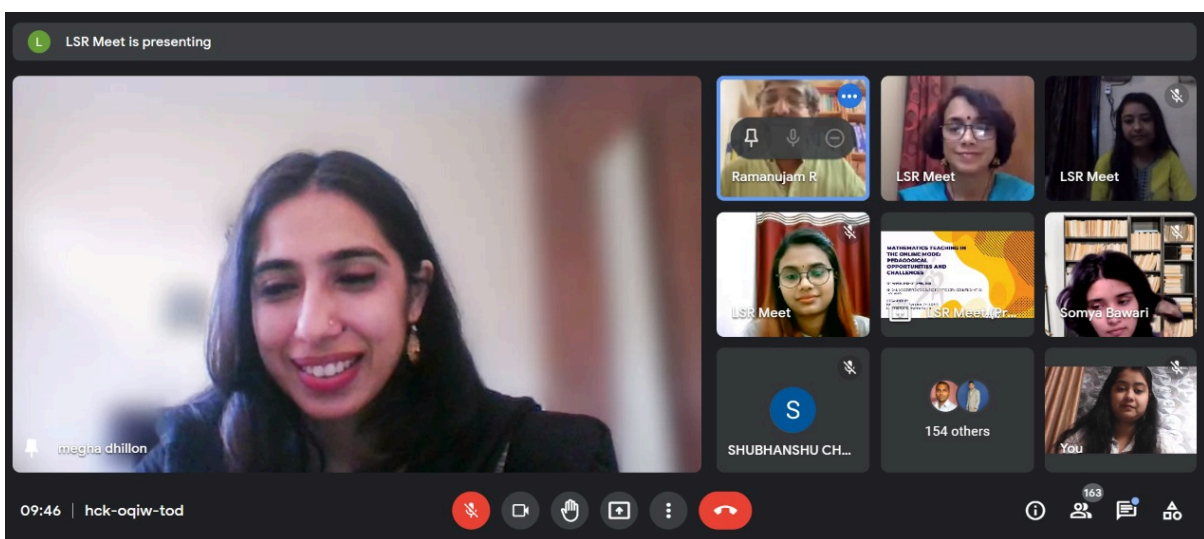
Central Institute of Education, DU
Maharshi Valmiki Institute of Education, DU
Jamia Millia Islamia
Shayama Prasad Mukherjee College, DU



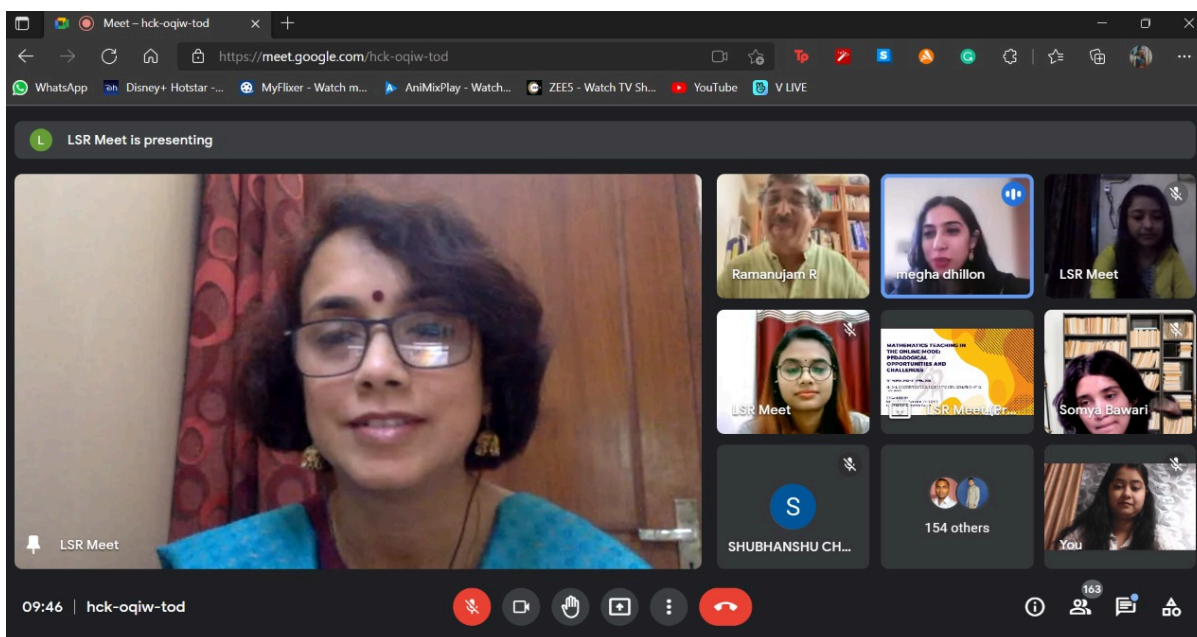
Inaugural session of the certificate course



Welcome address by Prof. Suman Sharma, Principal, Lady Shri Ram College for Women



Address by Dr. Megha Dhillon, IQAC Convener, Lady Shri Ram College for Women

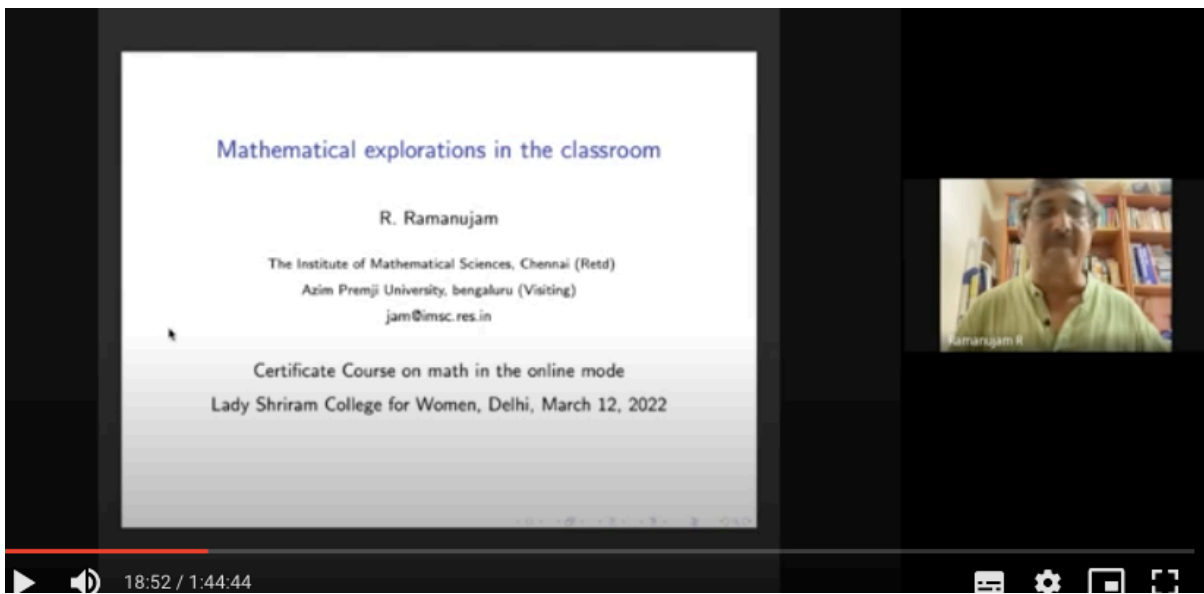


Introduction to the theme of the certificate Course by Dr. Jonaki B Ghosh, Certificate Course Coordinator, Lady Shri Ram College for Women

12th March 2022

Session 1 (10:00 – 11:30 am): Mathematical explorations in the classroom by Professor R Ramanujam, Institute of Mathematical Sciences Chennai (Retd.)

This was the opening session of the certificate course. Prof. Ramanujam highlighted the importance and role of mathematical explorations in teaching and learning of mathematical concepts. He illustrated children's thinking while engaging in explorations through various interesting examples. From playing with numbers and geometrical shapes to construction of Kolams, his session presented a wealth of ideas for incorporating exploratory tasks in the mathematics classroom. Drawing from the position paper on teaching of mathematics (NCF 2005), he emphasized the need for bringing about a shift from content to processes with regard to teaching and learning of mathematics. He reiterated the need to create opportunities to foreground processes such as estimation, approximation, visualisation, representation, reasoning, argumentation and making connections. He introduced the idea of Low Threshold High Ceiling (LTHC) tasks, which present multiple points of inquiry for students of all levels of ability. Towards the end of the talk he presented a guide map for mathematical explorations and discussed the characteristics of many styles of exploration, namely, question driven, concept driven and goal driven explorations. The talk was really eye opening and helped to set the stage for further discourse in the certificate course.

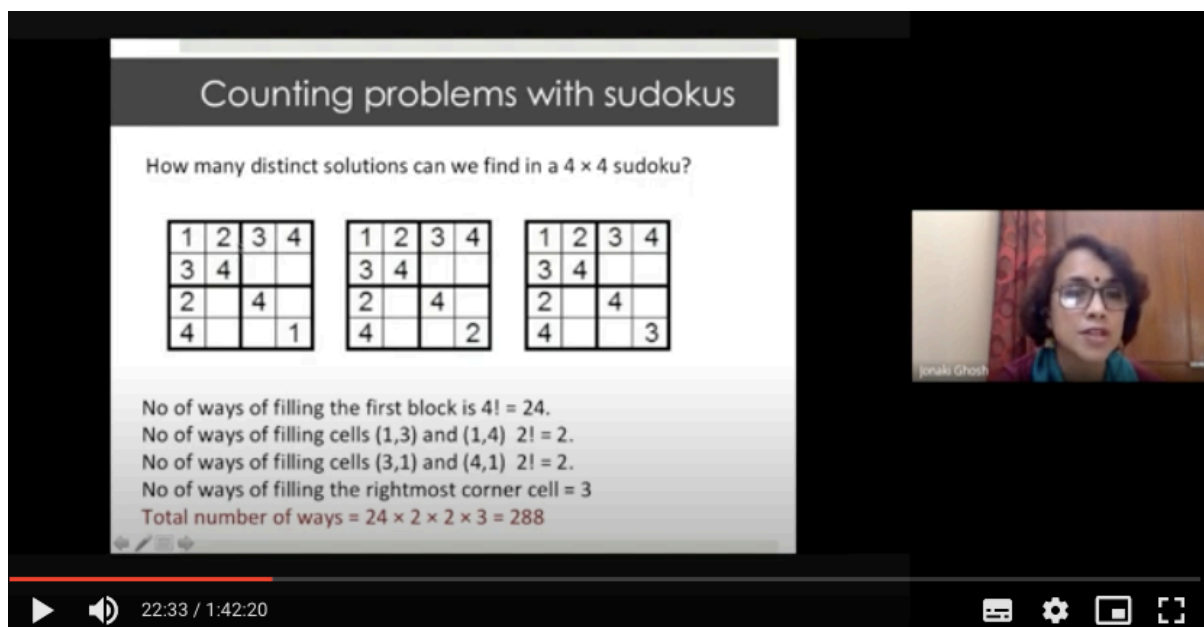


Photographs taken during the inaugural session of the certificate course By Prof. R Ramanujam

Session 2 (12:00 - 1:30 pm): Learning Mathematics through a spirit of inquiry

by Dr. Jonaki B Ghosh, Lady Shri Ram College for Women

Dr. Jonaki B Ghosh carried forward the ideas presented by Prof. Ramanujam. She began the session by highlighting the goals of mathematics education as articulated in the Position paper on teaching of mathematics (NCF 2005) and discussed the process of mathematisation and mathematical habits of mind. She engaged the participants in three exploratory problems. The first one was related to counting the number of distinct solutions of a 4 by 4 sudoku. This was a simple counting problem and helped to break the ice and engage the participants. The second problem was on counting squares and rectangles on a chessboard and generalizing the solution. The third problem was based on the Friendship Theorem. All the three problems saw the active participation of the audience and highlighted the opportunities provided by such problems for mathematical explorations.



Counting problems with sudokus

How many distinct solutions can we find in a 4×4 sudoku?

1	2	3	4
3	4		
2		4	
4			1

1	2	3	4
3	4		
2		4	
4			2

1	2	3	4
3	4		
2		4	
4			3

No of ways of filling the first block is $4! = 24$.
No of ways of filling cells (1,3) and (1,4) $2! = 2$.
No of ways of filling cells (3,1) and (4,1) $2! = 2$.
No of ways of filling the rightmost corner cell = 3
Total number of ways = $24 \times 2 \times 2 \times 3 = 288$

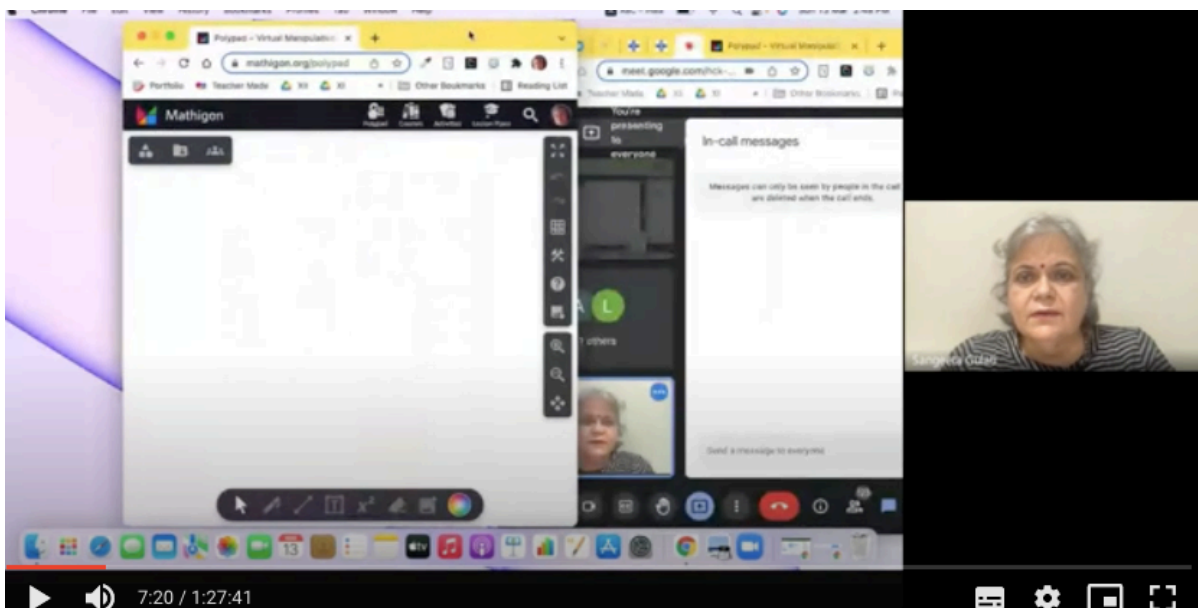
22:33 / 1:42:20

Session on Learning Mathematics through a spirit of inquiry by Dr. jonaki B Ghosh

Session 3 (2:30 – 4:00 pm): Getting started with GeoGebra

by Ms. Sangeeta Gulati, Head, Dept. of Mathematics, Sanskriti School

This was an introductory session on GeoGebra, an open source dynamic geometry software. Ms. Sangeeta Gulati illustrated all the basic features of GeoGebra, both as a computer software and also as a mobile phone app. This tool is extremely useful for online as well as offline classes and pre-service teachers need to be oriented to the use of GeoGebra as a tool for teaching mathematics. She presented many simple but interesting examples, such as construction of an equilateral triangle, use of sliders to create animations and using the GeoGebra Classroom feature for creating interactive tasks for students.

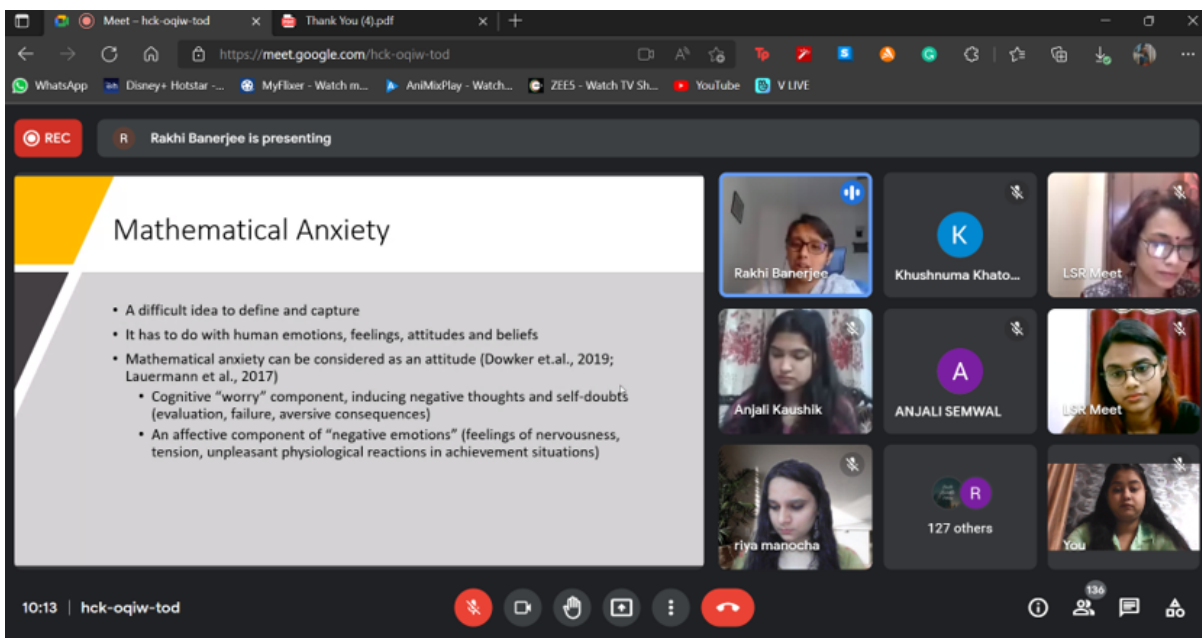
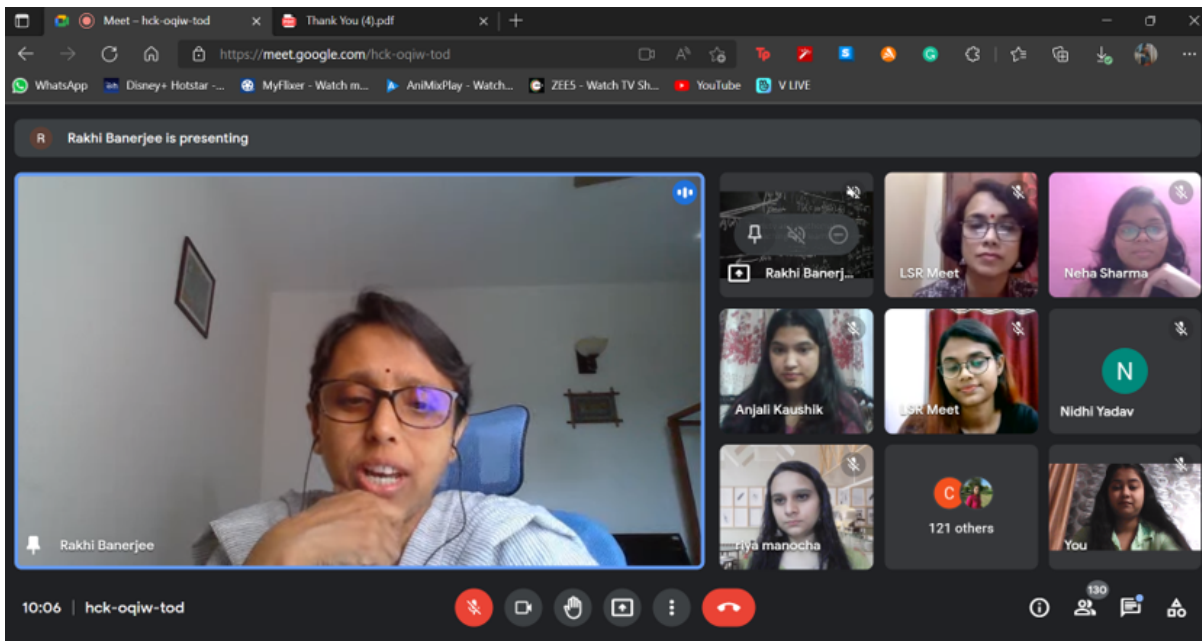


Hands on Introductory Session on GeoGebra by Ms. Sangeeta Gulati

13th March 2022

Session 4 (10:00 – 11:30 am): Anxiety and mathematics teaching and learning by Dr. Rakhi Banerjee, Azim Premji University

In this session, Dr. Rakhi Banerjee introduced the definition and causes of Mathematics Anxiety. She presented examples from her students' work, which focused on exploring the reasons for anxiety in children, such as learning gaps and timed achievement tests. She pointed out that the cultural context that links mathematics achievement with ability is a huge source of math anxiety. She focused on the conditions in teaching mathematics and the learning environment, which lead children developing math anxiety at a very early age. One important notion, which was highlighted in this session, contrary to popular belief, is that anxiety is always negative. One of the positive aspects is that it can also motivate the learner to strive to work harder. During the session a vibrant discussion ensued and many participants asked Dr. Banerjee about the consequences of math anxiety. She further went on to discuss Teacher's beliefs about mathematics and the consequences of math anxiety. Finally she proposed a few solutions as well. Overall the session was very informative and insightful for all the participants.

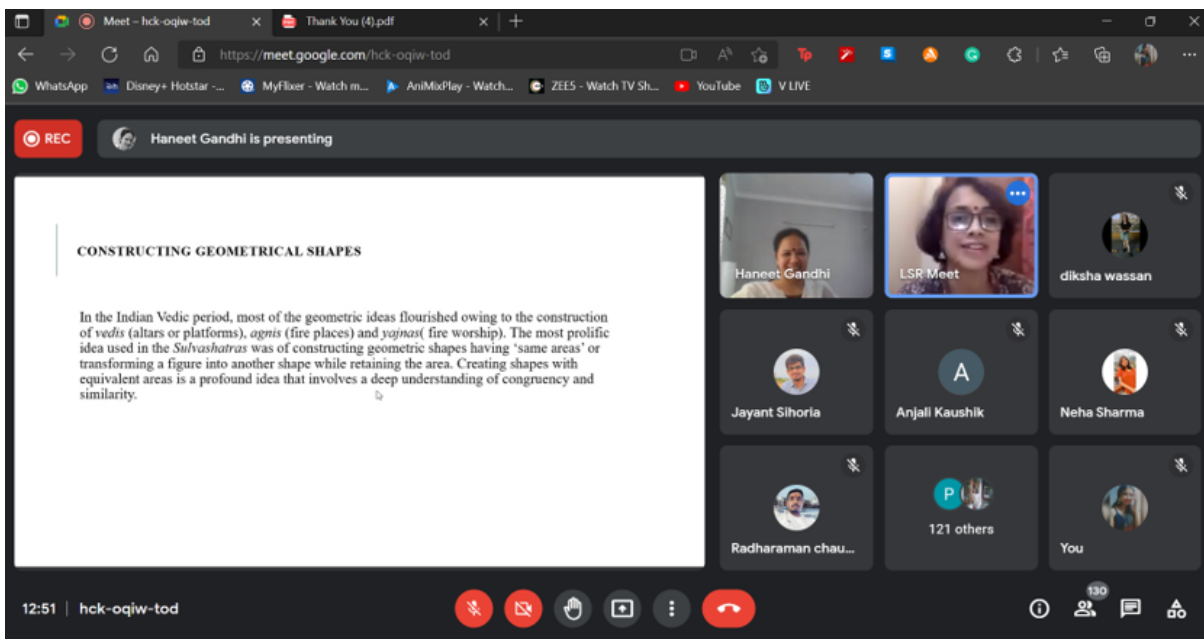
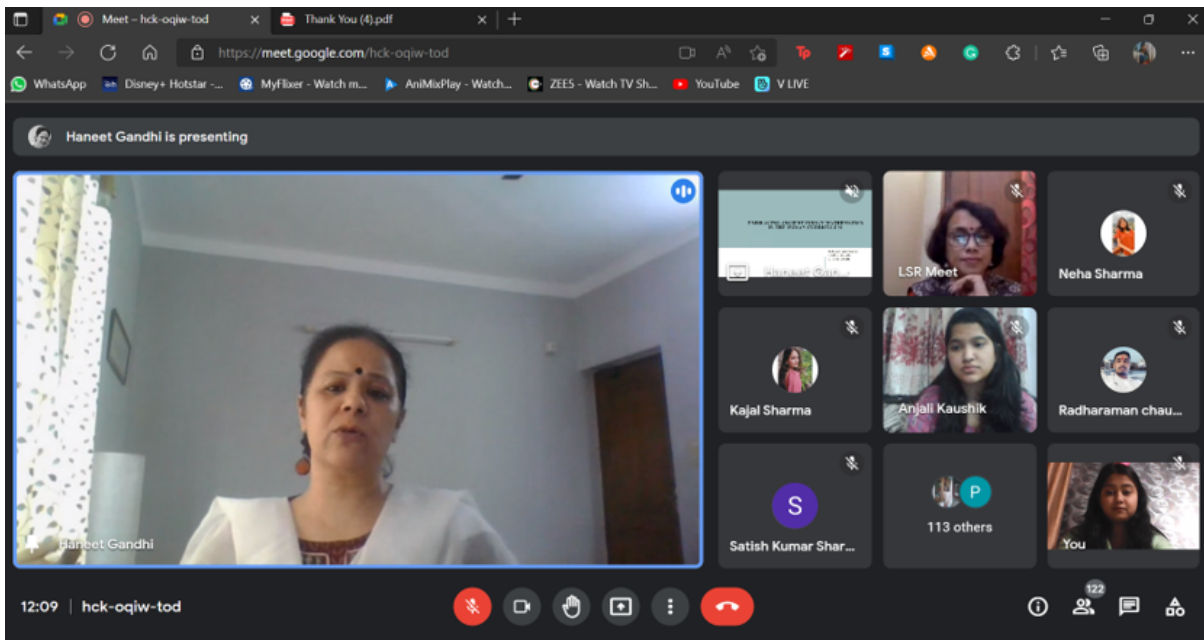


Session on Mathematics Anxiety by Dr. Rakhi Banerjee

Session 5 (12:00 – 1:30pm) : Embedding Indian Mathematics in the School Curriculum: Its Affordances and Challenges by Professor Haneet Gandhi, Central Institute of Education, Faculty of Education, University of Delhi

Prof. Haneet Gandhi highlighted the importance of ancient Indian mathematics and the need for remaining connected to our roots in a world, which is constantly being influenced by the western culture. She talked about how the textbooks and their content were redesigned based on the recommendations of NCF 2005. A great effort was made to ensure that the books remain connected to our country's rich heritage and culture in mathematics by including write ups of

noteworthy mathematicians and their notable works. The speaker mentioned the importance and need for introducing Vedic mathematics. She also mentioned that as soon as the political party changed, many debated that mathematics was getting sanskritized and was being wrongly introduced to the younger generations. The entire session was divided into two parts, first one highlighted glimpses of 'ancient Indian mathematics' and second part focused on gearing towards embedding ancient mathematics in our present curriculum. Various literature references were also provided by the speaker to make the session worthwhile and effective for the participants.



Session in Embedding Indian Mathematics in the School Curriculum by Prof. Haneet Gandhi

**Session 6: Mathigon (2:30 - 4:00 pm): Exploring with digital manipulatives
by Ms. Sangeeta Gulati, Head, Dept. of Mathematics, Sanskriti School**

This was an introductory session on Mathigon, an open source Virtual manipulative software. Ms. Sangeeta Gulati illustrated all the basic features of Mathigon, both as a computer software and also as a mobile phone app. This tool is extremely useful for online as well as offline classes and pre-service teachers need to be oriented to the use of virtual manipulatives as a tool for teaching mathematics. She presented many simple but interesting examples and highlighted how Mathigon can be integrated in primary as well as middle school teaching of mathematics. The session was very interesting and interactive.

16th March 2022

**Session 7 (3:00 – 4:30 pm): Mathematics in Social Practices
by Professor R Ramanujam, Institute of Mathematical Sciences Chennai (Retd.)**

In this session, Prof. Ramanujam gave us an opportunity to see social practices through a Mathematical lens. He has discussed how group preference is linked to logic and mathematics. He discussed the process of Elections at length and illustrated the role of mathematical logic in conceptualizing the process. The majoritarian rule is more prevalent in our society but there are other models as well which are followed in different countries. In Social choice theory, a dictatorship mechanism is a rule by which, among all possible alternatives, the results of voting, mirror a single predetermined person's preferences, without consideration to the preferences of other voters. He has also mentioned Amartya Sen's exemplary work on Social Choice Theory. His talk gave the audience insight into how we can use mathematics to talk about social practices and how we can effectively integrate them into our classroom teaching.



The screenshot shows a video lecture interface. The main content is a slide titled "Properties of Elections" with the following text:

A theory of elections suggests that there is much more.

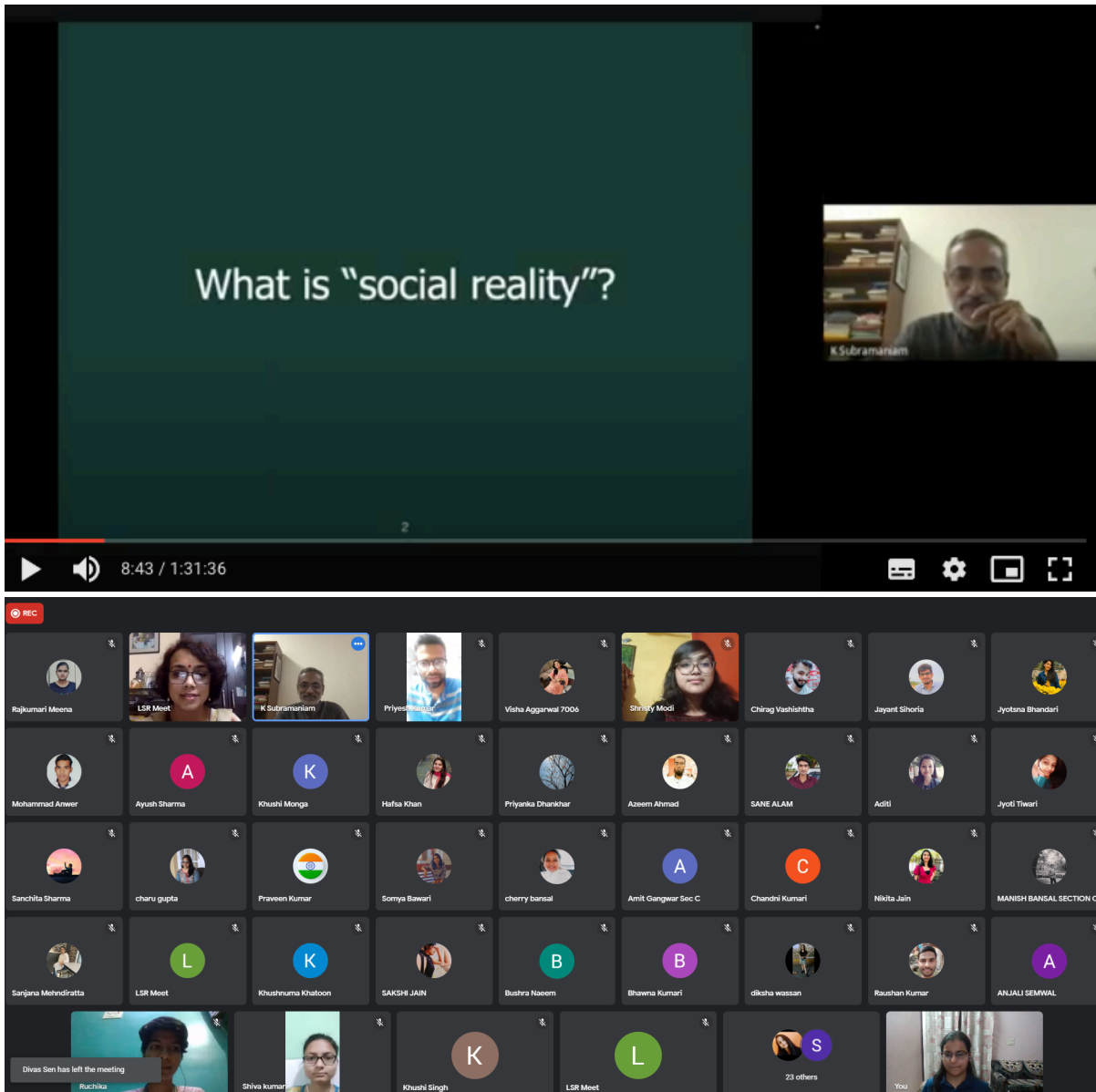
- ▶ **Universal verifiability:** we can check that every cast vote has been counted.
- ▶ **Individual verifiability:** every voter can check that *her* vote has been counted.
- ▶ **No summaries:** partial results are not available until all voters have voted.
- ▶ **Receipt freeness:** No voter can *prove* to another how he voted.

At the bottom of the slide, it says "Certificate course" and "March 16, 2022". The video player controls at the bottom show a play button, a volume icon, and a progress bar at 9:35 / 1:29:07. On the right side, there is a small video inset showing a man with a mustache, likely the speaker, in a room with bookshelves.

Session on Mathematics in social Practices by Prof. R Ramanujam

Session 8 (5:00 – 6:30 pm): Using data to understand social reality by Prof. K Subramaniam, Homi Bhabha Centre for Science Education, TIFR, Mumbai

In this session, Prof. K Subramaniam lent a deep insight into social reality and highlighted how misleading interpretations of data can keep us away from the truth. We tend to trust whatever is projected in the news and social media and often get fooled. He discussed trace data, which leaves traces of our activity when we travel from one place to another. He explained how sharing information on credit cards, mobile banking, and phone calls could be used and an unethical means of collecting data. Such data can be used to manipulate people's choices and preferences. Using examples of Kerala and Delhi, he demonstrated how Google mobility data could be used to interpret the movement behavior of people during COVID 19. He gave us a deep understanding of how data literacy helps us to understand social reality. His talk helped to create awareness that we should look at data critically and not become dependent on the media. He used actual data available from various sites to explore the meaning and information conveyed in terms of human behavior. The session was very interactive.



Session on Using data to understand social reality by Prof. K Subramaniam

23rd March 2022

Session 9 (3:00 - 4:30 pm): Developing Mathematical Thinking using Dynamic Software by Dr. Jonaki B Ghosh, Lady Shri Ram College for Women

Dr. Jonaki B Ghosh began the session by presenting a brief overview of the use of digital tools in mathematics education. Following this she highlighted the role of Dynamic Geometry Software, particularly GeoGebra, in enabling students' exploration of mathematical concepts. The emphasis was on using the features of GeoGebra, such as the dragging tool, for enabling students to explore concepts, generalize patterns and make conjectures. She focused on the learning theories such as the Variability Principle of Dienes, which inform the use of dynamic software in mathematics teaching. Through examples of students' work, she illustrated the fundamental differences in students' explorations of concepts in a dynamic geometry

environment and in paper-pencil mode. She illustrated the difference between drawing and construction and cited the results of a study in which grade 7 students constructed a square using GeoGebra. She ended the session with a discussion of the Nine Point Circle and illustrated how grade 9 students transitioned from conjecture making to proof through appropriate scaffolding. Overall the session was very interesting as it familiarized the audience with the use of GeoGebra as an *amplifier* and *reorganizer* of mental activity.

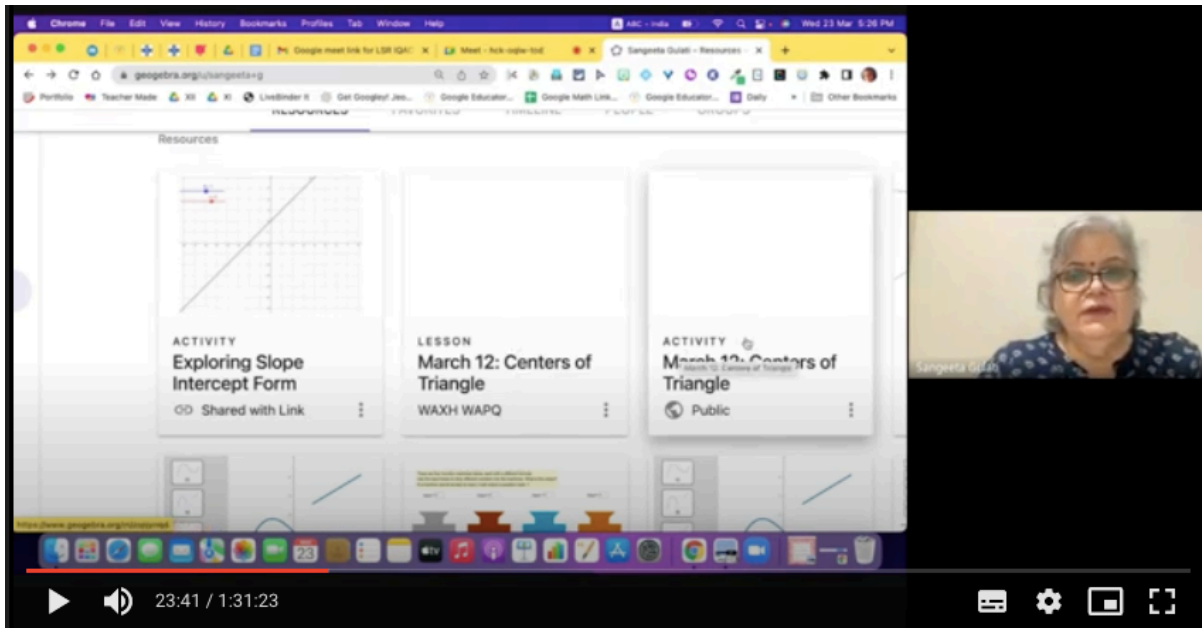
The Nine Point Circle Exploration: Conjecture making to proving by Grade IX students

In any triangle, the midpoints of the three sides (D,E,F), the feet of the three altitudes (I,J,K) and the midpoints of the segments joining the orthocentre to the vertices (L,M,N) - all nine points lie on a circle.

Hands on session on exploring mathematics using GeoGebra by Dr. Jonaki B Ghosh

Session 10 (5:00 – 6:30 pm): Creating interactive tasks in GeoGebra by Ms. Sangeeta Gulati, Head, Dept. of Mathematics, Sanskriti School

In this session, Ms Sangeeta Gulati demonstrated how different interactive tasks can be created using GeoGebra. She emphasized that GeoGebra can be used for exploring concepts in functions and pre-calculus. She explained the use of sliders for creating animations and also demonstrated how worksheets can be created and shared using GeoGebra classroom. This session along with session 3 of the certificate course provided the participants a very holistic understanding of the integration of GeoGebra as a tool for teaching and learning mathematics.



Hands on session on exploring mathematics using GeoGebra by Ms. Sangeeta Gulati

2nd April 2022

Session 11 (10:00 - 11:30 am) Mathematical explorations using spreadsheets by Dr. Jonaki B Ghosh, Lady Shri Ram College for Women

Spreadsheets can be very effective tools for exploring mathematical ideas and concepts. However, in comparison to dynamic software and other digital tools, they are generally overlooked and underutilized. Through various examples, the speaker illustrated that spreadsheets can be powerful in helping to identify patterns, in generating graphical representations and in creating simulations. In fact spreadsheets such as MS Excel or Libre Office Calc (or Google sheets) are very suited for the inquiry-based approach to learning and can be easily incorporated in online classes. She discussed four very interesting problems to demonstrate the manner in which Excel can be used to explore the problems and lend a deeper insight into their mathematical properties. The four problems, namely, the House number problem, The Birthday Paradox, The Spaghetti Problem and the Monty hall Problem were simulated in Excel and participants were encouraged to generalise patterns and explore solutions. The speaker emphasized that the beauty of spreadsheets lies in the fact that explorations, which are hard to do on paper and pencil, can be easily done on without the knowledge of high level of coding.

Spreadsheets for exploring mathematics

- ❑ Spreadsheets are generally overlooked and underutilised.
- ❑ Powerful and can help in identifying patterns, generating graphical representations and creating simulations.
- ❑ Very suited for the inquiry based approach to learning (and can be incorporated in online classes)
- ❑ High level of coding is not required.

1:22 / 1:34:29

S.No.	X	Y	a	b	c	Forms a triangle?
1	67	11	11	56	33	NO
2	34	53	34	19	47	YES
3	32	89	32	57	11	NO
4	8	91	8	83	9	NO
5	39	23	23	16	61	NO
6	22	27	22	5	73	NO
7	61	9	9	52	39	NO
8	41	21	21	20	59	NO
9	12	9	9	3	88	NO
10	45	94	45	49	6	YES
11	38	57	38	19	43	YES
12	45	66	45	21	34	YES
13	24	46	24	22	54	NO
14	11	26	11	15	74	NO

14:53 / 1:56:37

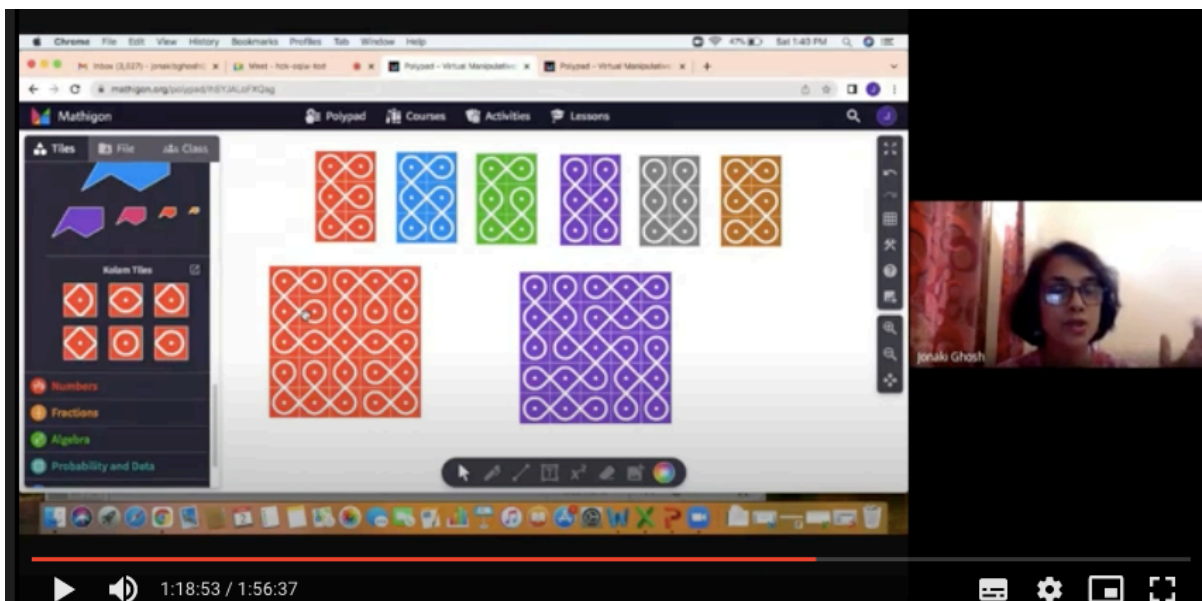
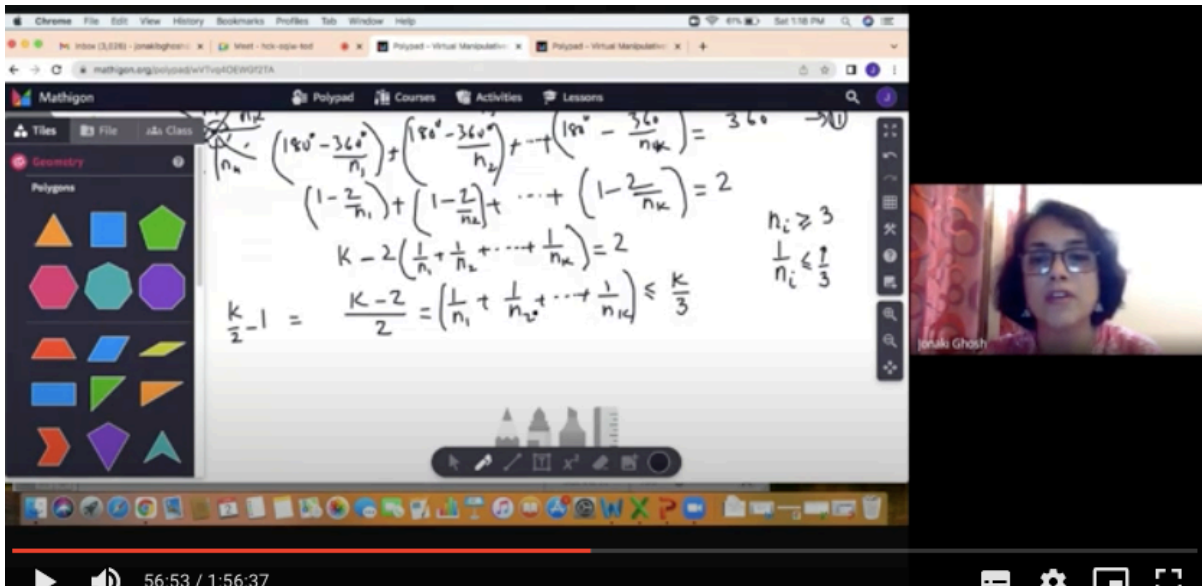
Hands on session on exploring mathematics using Spreadsheets by Dr. Jonaki B Ghosh

Session 12 (12:00 – 1:30 pm): Connecting Mathematics with Art

by Dr. Jonaki B Ghosh, Lady Shri Ram College for Women

Mathematics and art are inextricably connected. Dr. Jonaki Ghosh illustrated this with great alacrity in this session. In the first half she discussed the topic of tessellations and tiling and demonstrated the properties of regular and semi-regular tessellations using the virtual manipulative software, Mathigon. Participants were encouraged to explore the ideas using the polygonal tile feature of Mathigon. After discussing the various types of semi-regular

tessellations, these were enumerated mathematically and all possible vertex combinations were proved rigorously. In the second half of the session, she illustrated the properties and construction of kolams. Mathigon is equipped with kolam tiles and these can be used to construct complicated square and rectangular kolams. The construction of kolams is rooted in symmetry and this was demonstrated very nicely in this session. The speaker also highlighted that connection between kolams and group theory. This was the last session of the certificate course and was well received by the participants.



Hands on session on Art and mathematics using Mathigon by Dr. Jonaki B Ghosh

Valedictory Session (1:30 – 2:00 pm)

In the closing session of the certificate course four participants were invited to share their feedback and experience. The feedback was excellent and participants expressed a great sense of satisfaction as they felt that the course had introduced them to different aspects of mathematical pedagogy and content. They were particularly appreciative of the efficient way in which the course was conducted and lauded the efforts of the organizing team and IQAC, LSR. Dr. Jonaki B Ghosh presented the vote of thanks in which she thanked Principal, Prof. Suman Sharma for her encouragement and guidance in envisioning this certificate course and also in implementing it. She expressed her thanks to Dr. Megha Dhillon, Convener IQAC for her support and timely suggestions and also to Mr. Shailesh Kumar, Administrative officer, LSR and Mr. Sumit from the administrative department for providing the technical support. She expressed her sincere gratitude to all the experts who gave their valuable time in conducting the sessions of the course and briefly summarized each session. Finally, she expressed her thanks to the 4th year Pedagogy of Mathematics students of the Department of Elementary Education, for their tireless efforts in organizing the course. She mentioned the names of

- Ankita Gupta and Urvashi Rajput for contributing actively to the overall planning of the course, right from setting up the WhatsApp group, sending timely messages, handling participants' queries and also for managing the sessions held on 12th and 13th March.
- Riya Manocha, for designing the posters and certificates of the course, and for helping to coordinate the sessions on 12th March.
- Somya Bawari, for maintaining the attendance of each session and for compiling the same very efficiently. This was a critical and important task.
- Anjali Kaushik, for designing the powerpoint presentations which were used throughout the course at the beginning of each session. She also helped to anchor the sessions on 13th March.
- Stuty Jain, for efficiently managing to take pictures at the end of each sessions.
- Srishty Modi, for preparing the thank you notes for the speakers and for managing the sessions on 16th March.
- Neha Sharma, for managing the sessions on 13th March.
- Mehak Ahuja, for managing the sessions on 16th March.
- Sakshi Mandora and Aliya Qamar, for managing the sessions on 2nd April.

Finally, Dr. Jonaki Ghosh thanked all the participants for their active participation in the course.

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