Éclat

A Mathematics Journal

Volume I
2010

Department of Mathematics
Lady Shri Ram College For Women

Éclat

A Mathematics Journal

Volume **I**

2010

To
Our Beloved Teacher
Mrs. Santosh Gupta
for her 42 years of dedication

Preface

The revival of *Terminus a quo* into *Éclat* has been a memorable experience. *Éclat*, with its roots in french, means brilliance. The journey from the origin of *Terminus a quo* to the brilliance of *Éclat* is the journey we wish to undertake. In our attempt to present diverse concepts, we have divided the journal into four sections - History of Mathematics, Rigour in Mathematics, Extension of Course Contents and Interdisciplinary Aspects of Mathematics. The work contained here is not original but consists of the review articles contributed by both faculty and students.

This journal aims at providing a platform for students who wish to publish their ideas and also other concepts they might have come across. Our entire department has been instrumental in the publishing of this journal. Its compilation has evolved after continuous research and discussion. Both the faculty and the students have been equally enthusiastic about this project and hope such participation continues.

We hope that this journal would become a regular annual feature of the department and would encourage students to hone their skills in doing individual research and in writing academic papers. It is an opportunity to go beyond the prescribed limits of the text and to expand our knowledge of the subject.

Dootika Vats
Ilika Mohan
Rangoli Jain

Contents

History of Mathematics

The history behind various mathematical concepts and great mathematicians is intriguing. Knowing the history can lead to better development of concepts and enables us to understand the motivation behind ideas. Also, it shows us what inspired eminent mathematicians and highlights the problems they came across during their research.

Abelia - The Story Behind a Great Mathematician

Abstract

This paper is a summary of the life and works of the great 19th century mathematician Neil Henrik Abel(1802-1829). It is an attempt to elucidate the major incidents of his life, his significant achievements and also the legacy that he has left behind him.

INTRODUCTION

Neil Henrik Abel

Neil Henrik Abel was a noted Norwegian mathematician who has significantly contributed to the field of mathematics. His works are being studied currently by millions of aspiring mathematicians throughout the world.

CHILDHOOD

Abel was born in Nedstrand,Norway(near Finny) on 5th August, 1802, to Sren George Abel and Anne Marie Simonsen. Abel's father had a degree in theology and philosophy and his grandfather was an active protestant minister at Gjerstad near Risr. After the latter's death, Abel's father was appointed minister at Gjerstad. It was here that Abel grew up, together with his elder brother, three younger brothers and a sister.

EDUCATION

At the age of 13, Abel entered the Cathedral School of Christiania (today's Oslo). At the time when Abel joined the school it was in a bad state. This is because most of the good teachers had left the school in 1813 to join the newly established University of Christiania. The environment of the school failed to inspire Abel and he was nothing but an ordinary student with some talent for mathematics and physics.

Cathedral School In Cristiana

The school's mathematics teacher, Hans Peter Bader, had a reputation for conducting his classes in the old-fashioned way of copying from the blackboard. In 1817, he made the fatal mistake of beating a pupil to death. In protest against this horrendous act his students refused to attend any more of his classes until he was fired. The headmaster had to hastily find replacement and this event marked the entry of Bernt Michael Holmboe.

Holmboe was inspired by new pedagogical ideas. It was not long before he discovered young Abel's exceptional abilities and even gave Abel private tutoring and guided him .He even funded Abel's school education after his father's untimely death when his family was struck by poverty.

Bernt Michael Holmboe

A small pension from the state allowed Abel to enter Christiania University in 1821. In 1823 Abel had a chance to travel to Copenhagen to visit the mathematicians there, primarily Ferdinand Degen, who was regarded as the leading mathematician in the Nordic countries. In Copenhagen, Abel worked a little on Fermat's Great Theorem and elliptic functions. At a ball, Abel met the nineteen-year-old Christine Kemp, and was engaged to her.

Christine Kemp

Famous Works

During his short life, Abel devoted himself to several topics characteristic of the study of mathematics of his time. He chose subjects in pure mathematics rather than in mathematical physics. He worked on the following:

- solution of algebraic equations by radicals
- new transcendental functions, in particular elliptic integrals, elliptic functions, abelian integrals
- functional equations
- integral transforms
- theory of infinite series
- Proof of binomial theorem stated by Newton and Euler

Quintic Equation

We know that a fifth-degree equation (quintic equation) is of the form:

$$ax^5 + bx^4 + cx^3 + dx^2 + ex + f = 0$$

Though algebraic solutions for quadratic, cubic and quartic equations existed but no solutions for the quintic equation had been discovered so far. Abel eventually proved that an algebraic solution to the quintic equation was impossible. His work on quintic equations was his greatest work. Also, the adjective ”abelian”, used so frequently in mathematical writing has been derived from his name.

In Berlin Abel met a mathematically inclined engineer, August Leopold Crelle (1780-1855), who helped him publish his mathematical journal that could compete with the well-established journals in France. By the beginning of 1826, the first issue of Crelle’s Journal was published. Largely thanks to Abel’s works, Crelle’s Journal quickly became famous as one of Europe’s leading journals (the journal is still published today, and continues to have good international repute).

Memorabilia

Neil Henrik Abel contracted tuberculosis while working in Paris. He died at the age of 27 on 6th April, 1829 in Froland, Norway. After his death, Abel became a national hero in Norway. His birth centenary (1902) was widely celebrated and a number of memorials were erected-the most important among them was the monument by Vigeland which stands in the 'Abel Garden', the park of the Royal Palace. Few mathematicians have graced their country's stamps, banknotes and coins as often as Neil Henrik Abel.

Abel Prize

The prize was first proposed to be part of the 1902 celebration of the 100th anniversary of Abel's birth however the dissolution of the Union between Sweden and Norway in 1905 ended the first attempt to create the Abel Prize. In 2001, renewed interest in the prize resulted in the formation of a working group that developed a proposal to create this award. The first prize was awarded in 2003. The prize comes with a monetary award which, in 2009, was $9,29,000!

Abel Prize

S.R Shrinivasa Vardhan, an Indian working in the U.S. won the Abel Prize in 2007 for his exemplary work in probability theory and in particular for creating a unified theory of large deviation.

References

[1] John Stillwell, *Mathematics and its History*, 2nd Edition, pages 224-228
[2] Carl B. Boyer, *A History of Mathematics*, 2nd Edition, pages 583-588
[3] Jan Gullberg, *From the Birth of Numbers*, 1st Edition, pages 300-301

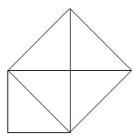
Aparna Krishnamurthy
Nikita Chaudhary
III year
Presented in Anupama Dua Paper Presentation

Geometry in Ancient Times

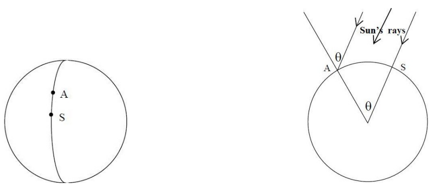
Abstract

The ancient civilizations of the Greeks, Egyptians and others were well aware of some well known mathematical results and used basic geometry concepts in order to measure great distances and construct monuments of massive sizes. This paper attempts to show just how well aware ancient civilizations were, even though unable to supply mathematical proofs.

The word geometry can be broken into two Greek words - 'geo' meaning earth and 'metry' meaning measurement. This makes geometry the science of systems involving measurement. The Greeks were obsessed with measuring distances. One such Greek was Plato (428 - 348 BC), who laid great emphasis on geometry, especially via observation. He justified his finding that the area of a square built on the diagonal of another was twice as big, simply by geometrical construction as shown below.

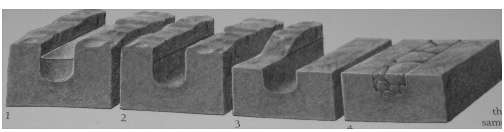


The Greeks were also aware that the Earth was not flat, and believed it to be roughly spherical. Eratosthenes, the polymath librarian of Alexandria, therefore sought to measure the circumference of the earth, in about 230 BC.



Eratosthenes assumed that the earth was perfectly spherical and that rays from the sun reaching the earth simultaneously were parallel. He felt these assumptions would not affect the answer greatly. The Nile ran from Syene to Alexandria. As Eratosthenes knew that the Nile ran roughly South-North, he thought of the part of the Nile flowing between the two towns as a part of a great circle - the circumference of the earth. The arc distance between Alexandria and Syene was known, around 5000 stadia. Now consider the angle θ subtended by this arc at the centre of the earth. Then AS : C = θ : 360, where C denotes the circumference of the earth. In order to find C, Eratosthenes had to find the angle θ. He knew that Syene had a very deep well whose water was touched by sunlight only at noon on the longest day of the year, so that at that point the sun would be directly over Syene. He realized that the angle θ subtended at the centre of the earth by the arc AS would be the same as the angle of the sun's inclination to the vertical in Alexandria when it was directly over Syene. So at noon one midsummer day he measured the sun's inclination to the vertical in Alexandria, and found it to be 7.2 degrees. Then by the above equation, he calculated C = (360/7.2) x 50 stadia = 50 x 5000 stadia = 250,000 stadia, or 46,250 km, which is just 6% off the actual circumference of 40,075 km.

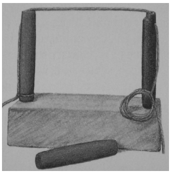
The Egyptians also used geometrical concepts in building the great pyramids, in the period 2600 BC. They knew how to measure area and planned the base area of the pyramids accordingly. They knew that a triangle with sides 3, 4, and 5 units yielded a right angle (Pythagoras Theorem), and used this to set the right angle corners of the pyramids. They used the same process to make the corners of the limestone blocks perfect right angles. They knew that water found its own level and used this to level the site where the pyramid would be built. A network of channels was dug across the site and filled with water. The level of water in the channels was marked and the water drained out. The channels were then cut to the line and filled with rubble to level them out.



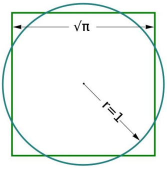
The Egyptians also used the concept of parallel lines to check if the limestone blocks cut were flat. They used wooden rods joined by strips of twine for this purpose. On the building site, blocks were also placed on wooden rockers so that they could move forward easily on the sand, minimizing friction.

They used the properties of the inclined plane in order to raise the blocks to great heights. Ramps were built which were raised along with the pyramid, layer by layer. As the ramp became higher, it became narrower. The Egyptians were also aware of the number π, and used it in calculations for construction.

Geometry was also studied in ancient India. Indians employed planning principles and proportions that rooted buildings to the cosmos and the movement



of the heavenly bodies. Everything that we know about ancient Indian Vedic mathematics is contained in the Sulbasutras, which are appendices to the Vedas, giving very specific rules for construction of fire altars, as unique altar shapes were associated with unique gods and purposes, like a rhombus for destroying enemies, etc. These also contain instructions for construction of basic geometrical shapes like rectangle, rhombus, etc and also included some complex constructions like transforming a square into a rectangle, an isosceles trapezium, an isosceles triangle. The most interesting problem recorded was that of 'circling a square' and 'squaring a circle'. In reality, it is impossible to exactly circle a square. i.e., construct a circle whose area is exactly the same as that of a given square, as this requires the construction of $\sqrt{\pi}$. But ancient India found an approximate solution to this problem, recorded by Apastamba (600 BC) in the Sulbasutras. The procedure for the circling a square is recorded as "If it is desired to transform the square into a circle, a cord of length half the diagonal of the square is stretched from the centre to the east (a part lying outside the region of the square), with one-third of the part lying outside added to the remainder of the half-diagonal, and then the required circle is drawn." The statement for squaring the circle is given as "Divide the diameter into fifteen parts and reduce it by two of them, which gives the approximate side of the required square." These give the value as 3.088 and 3.004 respectively.



In ancient Peru, the Incas constructed their walls with massive blocks, fitted together in precise mortar less joints. The stone blocks used were rarely square or even of uniform size. Yet each stone was fitted perfectly against the adjacent stone. Projections and indentations at the base were used for this purpose, as well as subtle chipping techniques. They were very particular about the angle at which the stones were chipped, in order to obtain a perfect fit. They used a medium sized hammer at an angle of 15-20 degrees from the vertical to chip off tiny flakes. The smallest hammer was used to chip the margins of each edge at an outward angle.

In medieval sieges, attackers used long range artillery in order to weaken the castle walls. Many of these were invented by the Greeks. The trebuchet and mangonel were used to hurl projectiles. They were constructed with longer and shorter arm lengths respectively in order to achieve a desired range for the projectile. The ram was a suspended tree trunk with an iron head, which was swung to and fro to hammer holes into castle walls and gates. Similar to this was the bore, a large metal spike. The Greeks, who invented it, knew that the impact would be greater that the tip of the metal spike.

In Babylonian Thales's mathematical knowledge was very advanced. He used geometry to solve problems such as calculating the height of pyramids and the distance of ships from the shore. The Babylonian's also used a sexagesimal number system to record and predict the positions of the Sun, Moon and planets.

We thus see that the ancient world was anything but ignorant of geometry. They inculcated it in almost everything they did. Geometry is indispensable to the understanding of structural concepts and calculations. It was also employed as visual ordering element - a means to achieve harmony.

References

[1] Reader's Digest,*How is it done?*, 1990, pages 319, 328, 338, 352, 367
[2] Marshall Clagett,*Ancient Egyptian Science*, 1995, pages 321 - 324
[3] Richard J.Gillings,*Mathematics in the time of the Pharaohs*, 1981
[4] *http://softsurfer.com/history.htm* (History of geometry)

H.B. Sahana
III year

Rigour in Mathematics

This section aims to introduce advanced topics in mathematics to students. It serves to stimulate interest in different branches of mathematics and lay the foundation for further study.

Lebesgue Measure and Integration

Abstract

In 1902, Henri Lebesgue presented an extension of the Riemann integral. Lebesgue's approach provided the ideal tool for research into troublesome issues not solvable by Riemann Integration. This paper discusses the basics of the concept of the "measure" of a set and how it is employed to overcome certain flaws with Riemann Integration, by using Lebesgue's approach to Intergration.

Riemann Integration and Its Drawbacks

We recall that if f is a bounded real valued function defined on the interval $[a, b]$ and $a = x_o < x_1 < x_2 ... < x_n = b$ is a partition of $P[a, b]$, then we can define:

$$S_P = \sum_{i=1}^{n}(x_i - x_{i-1})M_i \quad \text{and} \quad s_P = \sum_{i=1}^{n}(x_i - x_{i-1})m_i$$

where:
M_i = sup($f(x)$) for $x_{i-1} \leq x \leq x_i$
m_i = inf($f(x)$) for $x_{i-1} \leq x \leq x_i$

We then define the upper Riemann integral of f by:
$R\overline{\int} f(x)dx =$ inf$\{S_P$ — where P is a parition of $[a, b]\}$

Similarly the lower integral is defined as:
$R\underline{\int} f(x)dx =$ sup$\{S_P$ — where P is a parition of $[a, b]\}$

If the two are equal then we say that f is Riemann integrable and call this value the Riemann integral of f and denote it by $R\int_a^b f(x)dx$.

Another way of computing the Riemann integral of a function f is by approximating the area under the curve of the function by constructing a partition P of the domain $[a, b]$ and constructing a step function g approximating f and estimating the area under the graph of f by the area under the graph of the step function. This approximate area tends to a specific limit as the norm of the partition P tends to 0 and if the function f is Riemann integrable, i.e., we define:

$$g(x) = f(\xi_i) \quad \forall \quad x \in [x_{i-1}, x_i]$$

$$\text{where} \quad \xi_i \in [x_{i-1}, x_i] \quad \text{is arbitrarily chosen and fixed.}$$

and $S(f,P)=\sum_{i=1}^{n} f(\xi_i)l(x_{i-1},x_i),$ where $l(x_{i-1},x_i)=x_i-x_{i-1}$

$$\int_a^b f dx = \lim_{||P||\to 0}(S(f,P))$$

Interpretation: Essentially, Riemann integration involves computing the area under the curve of a function by calculating the area of rectangles under the curve by partitioning the domain.

Characterisation of Riemann Integrable Functions

It can be shown that all continuous functions are Riemann integrable. Although there are non continuous functions that are Riemann integrable(e.g., Step function), we will see that a function is Riemann Integrable iff it is *essentially continuous*.

Set of measure zero: A subset E of $\mathbb{R}$ is said to have measure zero if for each $\epsilon > 0$, there exists a sequence $\{I_n\}_n$ of open intervals such that $E \subset \bigcup_n I_n$ and $\sum_n l(l_n) < \epsilon$.

We now define an essentially continuous function as one whose set of points of discontinuity has measure zero.

Shortcomings of Riemann Integration

1) The class of Riemann integrable functions is relatively small.

   eg: Define

$$f(x)=I_Q=\begin{cases}0 & \text{if } x \text{ is irrational}\\ 1 & \text{if } x \text{ rational}\end{cases}$$

   Then, the lower integral of f(=0) and upper integral of f(=1) are not equal and hence f is not Riemann Integrable on [0,1] even though it is a bounded function.

2) Riemann integral does not have satisfactory limit properties. That is, given a sequence of Riemann Integrable functions $\{f_n\}$ with a limit function $f=\lim_{n\to\infty} f_n$. It does not necessarily follow that the limit function f is Riemann integrable. eg.:

Let $\{a_k\}$ be an enumeration of all the rationals in $[0, 1]$ (possible as rationals are countable).

Define $g_k(x) = \begin{cases} 1 & , x = a_j \leq a_k \\ 0 & \text{, otherwise} \end{cases}$

The function g_k is zero everywhere except on a finite set of points, hence its Riemann integral is zero. The sequence g_k is also clearly non-negative and monotonically increasing to I_Q, which is not Riemann integrable.

A NEW APPROACH: LEBESGUE INTEGRATION

An equally intuitive, but long in coming method of integration, was presented by Lebesgue in 1902. Rather than partitioning the domain of the function, as in the Riemann integral, Lebesgue chose to partition the range.

Thus, for each interval in the partition, rather than asking for the value of the function between the end points of the interval in the domain, he asked how much of the domain is mapped by the function to some value between the two end points in the range.

Partitioning the range of a function and counting the resultant rectangles becomes tricky since we must employ some way of determining (or measuring) how much of the domain is sent to a particular portion of a partition of the range. Measure theory addresses just this problem.

Measure Theory

Given an interval $E = [a, b]$ and a set S of subsets of E which is closed under countable unions and contains one empty set, we define the following: A set function μ on S is a function which assigns to each set $A \in S$ a real number and satisfies the following properties:

- Semi-Positive-Definite: $0 \leq \mu(A) \leq b - a \quad \forall \quad A \in S$.
- Trivial case $\mu(\phi) = 0$
- Monotonicity: $\mu(A) \leq \mu(B) \quad \forall \quad A, B \in S \quad \text{with } A \subset B$

- Countable Additivity: If $A = \bigcup_{n=1}^{\infty} A_n$ then
  $\mu(A) = \sum_{n=1}^{\infty} \mu(A_n)$ where $A_n \in S$ are pairwise disjoint for $n = 1, 2, ...$

We define **Lebesgue Outer Measure** of a subset E of $\mathbb{R}$ by:

$$m^*(E) = inf(\sum_{n=1}^{\infty}\{l(I_n) : E \subset \bigcup_{n=1}^{\infty} I_n\}$$

where the infimum is taken over all coverings of E by countable unions of intervals.

Interpretation: The outer measure of E is the infimum of certain *overestimates* of lengths of E since the sum of the length of non-disjoint intervals will be an overestimate of the length of their unions.
Properties:

1) $0 \leq m^*(E) \leq \infty$ for any E
2) If $E \subset F$, then $m^*(E) \leq m*(F)$
3) $m^*(E + x) = m^*(E) \;\; \forall \;\; x \in \mathbb{R}$
4) $m^*(E) = 0$ for any countable set E
5) $m^*(E) < \infty$ for any bounded set E
6) $m^*(I) = l(I)$ for any interval I, bounded or not
7) $m^*(E) = inf\{\sum_{n=1}^{\infty}(b_n - a_n) : E \subset \bigcup_{n=1}^{\infty}(a_n, b_n)\}$

In case of a collection of countably many pairwise disjoint open sets, m^* countable additive. However, there are cases where the countable additivity of m^* fails.

We can isolate a large class of sets on which m^* is countably additive. Specifically, we say that a set E is **(Lebesgue) measurable** if, for each $\epsilon > 0$, $\exists$ closed set F and an open set G with $F \subset E \subset G$ such that $m^*(G \sim F) < \epsilon$

It can be shown that the following sets are measurable:

- any interval, bounded or otherwise
- empty set
- a complement of measurable set
- any open or closed set
- union or intersection of countable number of measurable sets.
- difference of measurable sets

An example of a non measurable set is the Vitali Set:

Vitali Set: Axiom of Choice: 'For any set X of nonempty sets, there exists a choice function f defined on X' where, a choice function maps every set to one of its elements.

To construct a Vitali set V, consider the additive quotient group $\mathbb{R}/\mathbb{Q}$. Each element of this group is a "shifted copy" of the rational numbers: a set of the form $\mathbb{Q} + r$ for $r \in \mathbb{R}$. Thus, the elements of this group are subsets of $\mathbb{R}$ and partition $\mathbb{R}$. There are uncountably many elements in this group. Since each element intersects $[0, 1]$, we can use the axiom of choice to choose a set $V \subset [0, 1]$ containing exactly one representative out of each element of $\mathbb{R}/\mathbb{Q}$.

It can be shown that the Vitali set V, so constructed, is non-measurable.

Measurable Functions

Let A be a bounded measurable subset of $\mathbb{R}$ and $f : A \to \mathbb{R}$ be a function. Then f is said to be measurable on A if $\{x \in A | f(x) > r\}$ is measurable (as a subset of $\mathbb{R}$) for every real number r.
The function f is measurable if the shaded region of the domain is measurable as a subset of $\mathbb{R}$ for all choices of the real number r.

Integrating Bounded Measurable Functions

Let $f : A \to \mathbb{R}$ be a bounded measurable function on a bounded measurable subset A of $\mathbb{R}$. Let $l = inf\{f(x)|x \in A\}$ and $u > sup\{f(x)|x \in A\}$ where u is arbitrary in so far as it is greater than the least upper bound of f on A.

As with the Riemann integral, we'll define the Lebesgue integral of f over an interval A as the limit of some *Lebesgue sum*.

The Lebesgue sum of f with respect to a partition $P = \{l = y_0 < ... < y_n = u\}$ of the interval $[l, u]$ is given as :

$$L(f, P) = \sum_{i=1}^{n} y_i^* m^* \{x \in A \mid y_{i-1} \leq f(x) \leq y_i\}$$

where $y_i^* \in [y_{i-1}, y_i]$ for $= 1, ..., n$ and f is a bounded measurable function over a bounded measurable set A of $\mathbb{R}$.

This is the new way to count rectangles; the y_i^* is the height of the rectangle and the $m(\{x \in A \mid y_{i-1} \leq f(x) \leq y_i\})$ serves as the base of the rectangle. The definition of the actual Lebesgue integral is virtually identical to that of the Riemann integral using sum function.

A bounded measurable function $f : A \to \mathbb{R}$ is Lebesgue integrable on A if there is a number $L \in \mathbb{R}$ such that, given $\epsilon > 0 \exists$ a $\delta > 0$ such that $|L(f, P) - L| < \epsilon$ whenever $||P|| < \delta$. $L$is known as the Lebesgue integral of f on A and is denoted by $\int_A f dm$

Simple Functions: A simple function is a finite linear combination of indicator functions of measurable sets. More precisely, let (X, Σ) be a measurable space. Let $A_1, ..., A_n$ be a sequence of measurable sets, and let $a_1, ..., a_n$ be a sequence of real or complex numbers. A simple function is a function of the form:

$$\varphi(x) = \sum_{k=1}^{n} a_k I_{A_k}(x)$$

Lebesgue Integration of Simple Functions

For a simple function φ defined on $\mathbb{R}$ as above, is:

$$\int \varphi = \int_{\mathbb{R}} \varphi = \int_{-\infty}^{\infty} \varphi(x) dx = \sum_{i=1}^{n} a_i m(A_i)$$

Advantages of Lebesgue Integration

- Limit of a sequence of Lebesgue integrable functions is also Lebeague integrable.

- Class of Lebesgue integrable functions is larger than that of Riemann integrable functions. eg.:
  Consider:
  $$f(x) = \begin{cases} 0 & x \text{ is irrational} \\ 1 & x \text{ is rational} \end{cases}$$

then, $f(x) = I_Q$, therefore, $\int f = 1.m(\mathbb{Q}) = 1.0 = 0$

Hence, we can see that Lebesgue's approach to integration is a useful tool for mathematicians and the basis for many branches of mathematics.

References

[1] N.L Carothers, *Real Analysis*, 2006, pages 263-335
[2] H.L. Royden, *Real Analysis*, 2004, pages 54-96
[3] Joshua H. Lifton, *Measure Theory and Lebesgue Integration*, 1999

Rushil Prakash
III Year

Construction of the Real Number System

Abstract

In this paper we show that $\mathbb{R}$ is the only complete Archimedean ordered field. We give two approaches to completeness for Archimedean ordered fields and conclude that they are equivalent.

Defect in Rationals

Although we have noticed that the set of rationals $\mathbb{Q}$ forms a rich algebraic system having order properties, it is inadequate for the purpose of analysis. We have already noticed that not every positive rational number has a rational square root. For example we have seen that there is no rational number whose square is 2. The defect in the rationals which we wish to describe here may be described in a variety of ways. One form of the defect that we have already noticed is that a nonempty subset of rational numbers that is bounded above need not have a least upper bound (in the set of rationals). A slighty less standard approach which is more picturesque is that there are open lower segments in $\mathbb{Q}$ without having right end points in $\mathbb{Q}$, and we consider this to be a defect. For this purpose, we define the notion of an open lower segment in the set of rationals.

Definition 1:A set $J \subset \mathbb{Q}$ is called an open lower segment if

1) $J \neq \emptyset$,
2) $J \neq \mathbb{Q}$,
3) For every $x \in J$, there is a $y \in J$ such that $y > x$,
4) If $x \in J$ and $y < x$ then $y \in J$.

A point x will be called the right end point of an open lower segment J if

1) for every $y \in J$, $x > y$,
2) if z is such that for every $y \in J$, $z > y$, then $z \geq x$.

For example, consider $J \subset \mathbb{Q}$ to be the set of all non-positive rational numbers, together with the positive rational numbers whose square is less than 2. Then J is an open lower segment of $\mathbb{Q}$ having no right end point in $\mathbb{Q}$

Note. If a right end point of J exists, it is unique, and is the smallest rational greater than every element of J.

The Real Numbers

The set of real numbers is an extension of the set of rational numbers, which removes the defect described above. We will see that the real numbers are an Archimedean ordered field in which every open lower segment has a right end point. The real numbers are obtained from the rationals by what may be described as filling the gaps. In other words we add right end points as ideal elements, or new numbers, to correspond to those open lower segments which do not have right end points among the rationals.

One way of doing this is to consider the open lower segment itself to be a substitute for its own right end point. One can see that this is an entirely natural approach when one agrees that open lower segments are to be in one-to-one correspondence with their right end points.

In accordance with the above, the set $\mathbb{R}$ of real numbers is defined to be the set of open lower segments of rational numbers.

We first define addition in $\mathbb{R}$.

Let I and J be real numbers i.e., open lower segments of rational numbers. Define $I+J$ as

$$I+J=\{x+y : x\in I, y\in J\}$$

We show that $I+J$ is an open lower segment.

Let $x\in I$ and $y\in J$, so that $x+y\in I+J$, and let $u<x+y$. Then we can write $u=x+z$, where $x\in I$ and $z(=u-x)<y$, so that $z\in J$. So, $u\in I+J$. Next there is a $z>y$ such that $z\in J$. So, $x+z>x+y$ and $x+z\in I+J$. Now, it follows that $I+J$ is an open lower segment.
We have

$$I+J=J+I$$

and that

$$(I+J)+K=I+(J+K)$$

follows directly from the definition of $'+'$ in $\mathbb{R}$ and commutative and associative properties of $\mathbb{Q}$.
Let O be the open lower segment of negative rational numbers. One can verify that

$$I+O=I \quad \forall \ I\in\mathbb{R}$$

Finally, the equation

$$I+X=J$$

has a solution for every $I, J \in \mathbb{R}$. We simply let X consist of all x such that $y + x \in J$ for every $y \in I$, except for the largest such x if there is one. The solution of $I + X = O$ is designated $-I$.

An order relation is introduced in $\mathbb{R}$ by letting

$$I > J \quad \text{if} \quad I \supset J$$

Thus $\mathbb{R}$ becomes an ordered set. Clearly $I > O$ if and only if I contains a positive rational.

We now define multiplication in $\mathbb{R}$. If $I > O, J > O$ we define IJ as the set of all nonpostive rationals together with all xy where $x \in I$, $x > 0$, and $y \in J$, $y > 0$

If $I = O$ or $J = O$ we define $IJ = O$. If both $I < O$ and $J < O$ we define

$$IJ = (-I)(-J)$$

If exactly one of I, J is less than O, say $I < O$, we define

$$IJ = -(-I)(J)$$

Now one can show that $\mathbb{R}$ is an ordered field. The complete details can be found in [2].

Moreover, $\mathbb{R}$ is Archimedean. Let $I > O$, $J > O$. Then there exists a positive rational $x \in I$ and an $n \in \mathbb{N}$ such that $nx \notin J$, since $\mathbb{Q}$ is Archimedean and I, J are open lower segments. Thus, it follows from the trichotomy property of $\mathbb{R}$ that $nI > J$.

Then $J \in \mathbb{R}$ for which J has a rational right end point are in one-to-one correspondence with the rational numbers, the associated mapping is order-preserving and addition and multiplication preserving.

Thus, $\mathbb{Q}$ is imbedded in $\mathbb{R}$, or that $\mathbb{Q}$ is isomorphic to an ordered subfield of $\mathbb{R}$. We call this subfield the rational numbers and hence will use small letters for elements of $\mathbb{R}$ in Theorem 2.
We now define open lower segments of reals and their right end points in the same way as for rationals.

Theorem 1: Every open lower segment of reals has a right end point.

Proof: Let $\mathcal{I}$ be an open lower segment of reals. Let

$$U = \bigcup \{J : J \in \mathcal{I}\}$$

We show that U is an open lower segment of rationals i.e., $U \in \mathbb{R}$. Let $x \in U$. Then there is a $J \in \mathcal{I}$ such that $x \in J$. For every $y < x, y \in J \subseteq U$. Also there is a $y \in J \subseteq U$ such that $y > x$. Moreover, there is an $I \notin \mathcal{I}$ since $\mathcal{I}$ is an open lower segment of reals. Then $x \notin I$ implies $x \notin U$ because if $x \in U$, then $x \in J$ for some $J \in \mathcal{I}$ which implies $I \subset J$ and hence $I \in \mathcal{I}$, a contradiction. Thus U is an open lower segment of rationals. We now show that U is the right end point of $\mathcal{I}$. By definition of U, $U > J$ for every $J \in \mathcal{I}$. Suppose $V > J$ for every $J \in \mathcal{I}$. Then $V \supset U$, so that $V \geq U$. Hence U is the right end point of $\mathcal{I}$

.

We give another form of Theorem 1 which is referred to as the least upper bound property (or completeness property in th sense of Dedekind) of the set of real numbers $\mathbb{R}$

Theorem 2: If $S \subset \mathbb{R}$ is nonempty and has an upper bound, it has a least upper bound.

Proof: If the given upper bound belongs to S then that will only be the least one and we are done. So we assume that no upper bound for S is in S. Now we define a set U by letting $x \in U$ if and only if there is a $y \in S$ such that $y > x$. We show that U is an open lower segment of reals. If $x \in U$ and $z < x$ then $z \in U$ by definition of U. Also, if $x \in U$ then there is a $z \in U$ such that $z > x$ since no upper bound for S is in S. Moreover, every upper bound of S is not in U. Thus, U is an open lower segment of reals, and so it has a right end point,say u by Theorem 1. We show that $u = \sup S$.

Let $x \in S$. Then $y < x$ implies $y \in U$ so that $y < u$. Hence $x \leq u$ (since $u < x \Rightarrow u \in U$ which is a contradiction). Thus, u is an upper bound of S. Let $y < u$. Then $y \in U$ since U is an open lower segment and u is its right end point. So there is an x in S with $y < x$. Thus y is not an upper bound of S. Hence $u = \sup S$.

Thus it is actually the property of Theorem 1 that is being used to obtain all further properties of the real numbers. Thus, the fact that $\mathbb{R}$ is a complete Archimedean ordered field is basic to all further developments.

The Theorem 3 below shows that $\mathbb{R}$ is the only complete Archimedean ordered field.

Theorem 3: Any two complete Archimedean ordered fields F_1 and F_2, with sets of positive elements $P_1$and P_2, respectively, are algebraically and order isomorphic, i.e., there exists a one-to-one mapping τ of F_1 onto F_2 such that

$$\tau(x + y) = \tau(x) + \tau(y),\ \tau(xy) = \tau(x)\tau(y),\ \tau(x) \in P_2 \text{ iff } x \in P_1.$$

Proof: Let 1_1 and 1_2 be the units of F_1 and F_2 and 0_1 and 0_2 the zeros. Note that every ordered field contains an isomorph of $\mathbb{Q}$. So we define the mapping τ first on the rational elements of F_1 as follows:

$$\tau(\tfrac{m}{n}1_1) = \tfrac{m}{n}1_2$$

where m is an integer, n is a nonzero integer

If $x \in F_1$ and x is not of the form $\frac{m}{n}1_1$, then we define

$$\tau(x) = \sup\{\frac{m}{n}1_2 : \frac{m}{n}1_1 < x\}$$

One can prove that τ has the desired properties.

We now give another construction of the real number system in which a real number is defined as an equivalence class of Cauchy sequences of rational numbers.

Definition 2: Let F be an ordered field. A sequence (a_n) of elements of F is called bounded if there is an element $b \in F$ such that $|a_n| \leq b$ for each positive integer n.

Definition 3: A sequence (a_n) of elements of F is called Cauchy if for every $e \in F$ such that $e > 0$, there is a positive integer N such that $|a_p - a_q| < e$ for all $p, q \geq N$.

Definition 4: A sequence (a_n) of elements of F is called null if for every $e \in F$ such that $e > 0$, there is a positive integer N such that, $|a_p| < e$ for all $p \geq N$.

The families of sequences satisfying these conditions will be denoted by $\mathcal{B}$, $\mathcal{C}$ and $\mathcal{N}$ respectively.

We now state few theorems and lemmas (without proofs) before stating the two important results which concern the main theme of this paper. The details of the proofs can be found in [3].

Theorem 4: The inclusions $\mathcal{N} \subset \mathcal{C} \subset \mathcal{B}$ is obtained.

Theorem 5: For (a_n), $(b_n) \in \mathcal{C}$, let $(a_n) + (b_n) = (a_n + b_n)$ and $(a_n)(b_n) = (a_n b_n)$. With these definitions of sum and product, $\mathcal{C}$ is a commutative ring with unity, and $\mathcal{N}$ is an ideal in $\mathcal{C}$ such that $\mathcal{N} \subsetneq \mathcal{C}$.

Theorem 6: Let $\mathcal{C}/\mathcal{N}$ denote the set whose elements are the sets $(a_n) + \mathcal{N}$ (called cosets of $\mathcal{N}$), where $(a_n) \in \mathcal{C}$. Addition and multiplication in $\mathcal{C}/\mathcal{N}$

are defined by

$$((a_n)+\mathcal{N})+((b_n)+\mathcal{N})=(a_n)+(b_n)+\mathcal{N}=(a_n+b_n)+\mathcal{N} \text{ and}$$

$$((a_n)+\mathcal{N})((b_n)+\mathcal{N})=(a_n)(b_n)+\mathcal{N}=(a_nb_n)+\mathcal{N}$$

These definitions are unambiguous, and with addition and multiplication so defined, $\mathcal{C}/\mathcal{N}$ is a field

Notation: The field $\mathcal{C}/\mathcal{N}$ will be written as $\bar{F}$. Henceforth elements $(a_n)+\mathcal{N}$ of $\mathcal{C}/\mathcal{N}$ will be denoted by small greek letters: $\alpha, \beta, ...$. If $a \in F$ then the element $(a_n)+\mathcal{N}$ of $\bar{F}$ will be written as $\bar{a}$; it is the coset of $\mathcal{N}$ containing the constant sequence all of whose terms are a.

Theorem 7: In $\bar{F}$, let $\bar{P}=\{\alpha \in \bar{F} : \alpha \neq \bar{0}$ and there exists $(a_n) \in \alpha$ such that $a_n > 0$ for $n=1,2,...\}$ With this set $\bar{P}$, $\bar{F}$ is an ordered field. The mapping $\tau : \tau(a)=\bar{a}$ is an order preserving algebraic isomorphism of F into $\bar{F}$.

Definition 5: Given a sequence (a_n) in an ordered field F and $b \in F$, we say that limit of (a_n) is b and we write

$$\lim_{n\to\infty} a_n = b \quad \text{or} \quad a_n \to b$$

if for every positive e in F there exists a positive integer L such that $|a_n - b| < e$ for all $n \geq L$. An ordered field is said to be complete (in the sense of Cantor) if every Cauchy sequence in F has a limit in F.

Lemma 1: A sequence with a limit is a Cauchy sequence. If (a_n) is a Cauchy sequence and (a_{n_k}) is a subsequence with limit b, then (a_n) has limit b.

Lemma 2: For $\alpha > 0$, $\alpha \in \bar{F}$, there exists $e \in F$ such that $\bar{0} < \bar{e} < \alpha$. If F is Archimedean ordered, then $\bar{F}$ is also Archimedean ordered.

Lemma 3: Let $\alpha \in \bar{F}$ and $(a_n) \in \alpha$. Then we have

$$\lim_{n\to\infty} \bar{a}_n = \alpha$$

We now state the two important results mentioned above.

Theorem 8: The field $\bar{F}$ is complete (in the sense of Cantor).

The following Theorem 9 below shows that a complete Archimedean ordered field (complete in the sense of Cantor) is also complete Archimedean ordered field (complete in the sense of Dedekind).

Theorem 9: Let F be a complete Archimedean ordered field (complete in the sense of Cantor), and let A be a non empty subset of F that is bounded above. Then $\sup A$ exists.

Note: Theorem 3 above also holds for complete Archimedean ordered fields (complete in the sense of Cantor).

So, we have the following definition:

Definition 5: The real number field $\mathbb{R}$ is any complete ordered field. For example,$\bar{\mathbb{Q}}$.

Conclusion: We see that the two approaches to completeness for Archimedean ordered fields are equivalent. So for the reals it is entirely a matter of choice which approach one prefers. However, there are situations in which the Cauchy sequence approach is the only one possible.For example, in the field of complex numbers $\mathbb{C}$ which is not ordered ($\because\ \iota \neq 0$ but $\iota^2 = -1 < 0$) Theorem 8 which is another version of the completeness property for fields does not require the order relation, $<$. It is a useful axiom to consider for other fields other than ordered fields. All that is required is the distance function d(x,y) to have meaning in that field .

References

[1] Casper Goffman, *Introduction to Real Analysis*
[2] W. Rudin, *Principle of Mathematical Induction*
[3] Hewitt and Stromberg, *Real and Abstract Analysis*
[4] N.L. Carothers, *Real Analysis*

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Fuzzy Logic in Action

Abstract

This paper introduces Fuzzy logic which is a form of multi-valued logic derived from fuzzy set theory to deal with reasoning that is approximate rather than precise. The fuzzy logic variables have a membership value between 0 and 1, that is, the degree of truth of a statement is in the range of 0 and 1 and is not constrained to the two truth values of classic propositional logic. Fuzzy logic has been applied to many fields, from control theory to artificial intelligence.

FUZZY SET

Fuzzy sets are the sets whose elements have degrees of membership. Fuzzy sets were introduced by Lotfi A. Zadeh (1965) as an extension of the classical notion of set. In classical set theory, the membership of elements in a set is assessed in binary terms according to a bivalent condition - an element either belongs or does not belong to the set. By contrast, fuzzy set theory permits the gradual assessment of the membership of elements in a set. This is described with the aid of a membership function valued in the real unit interval [0, 1]. The fuzzy set theory can be used in a wide range of domains in which information is incomplete or imprecise, such as bioinformatics.

DEFINITION

Let X be some set of objects, with elements noted as x. A fuzzy set A in X is characterized by a membership function $mA : X \rightarrow [0.0, 1.0]$. As $mA(x)$ approaches 1.0, the "grade of membership" of x in A increases.

- x is called 'not included' ; if $mA(x) = 0.0$,
- x is called 'fully included' ; if $mA(x) = 1.0$,
- x is called 'fuzzy member' ; if $0.0 < mA(x) < 1.0$.
- $A = B$ iff for all $x : mA(x) = mB(x)$ [or, $mA = mB$].
- $mA' = 1 - mA$.

- $A \subset B$ iff $mA \leq mB$.
- C = $A \cup B$, where: m$C(x) = max(mA(x), mB(x))$.
- C = $A \cap B$ where: m$C(x) = min(mA(x), mB(x))$.

UNDERSTANDING FUZZY

To understand the concept of fuzzy, we consider an example.

Suppose, a question is posed: 'Is that person over 180 cm feet tall?'. This question has only two answers: YES or NO. On the other hand, the question 'Is that person tall?' has many answers. Someone over 190 cm is universally considered to be tall. Someone who is 180 cm may be considered to be sort of tall, while someone who is under 160 cm is not usually considered to be tall.

The graph has a value 0 for any value under 160 (people under 160 cm are not tall) and a value of 1 for any value over 190 cm (people who are over 190 cm are tall) and varying degrees of membership in the set of people who are tall for values between 160 and 190. This graph is called the membership function of the fuzzy set of people who are tall.

Consider another statement: "Jane is old."

If Jane's age was 75, the statement could be translated into set terminology as follows:
"Jane is a member of the set of old people."
This statement would be rendered symbolically with fuzzy sets as:

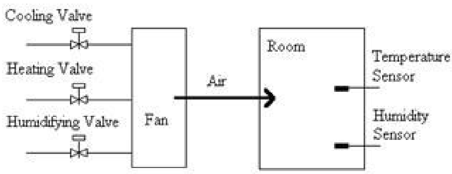
$$mOLD(Jane) = 0.80\ (say)$$

Where m is the membership function, operating in this case on the fuzzy set of old people, which returns a value between 0.0 and 1.0. As we notice, fuzzy systems and probability are quite similar. Both operate over the same numeric range, and at first glance both have similar values. However, there is a distinction to be made between the two statements. The probabilistic approach yields the natural-language statement,"There is an 80% chance that Jane is old," while the fuzzy terminology corresponds to "Jane's degree of membership within the set of old people is 0.80". The difference is significant: the first view supposes that Jane is or is not old; it is just that we only have an 80 chance of knowing which set she is in. By contrast, fuzzy terminology supposes that Jane is ”more or less” old, or some other term corresponding to the value of 0.80

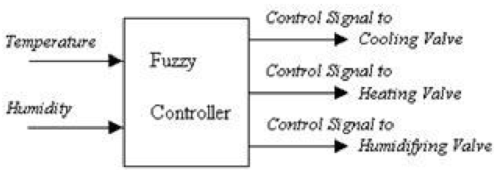
APPLICATION OF FUZZ THEORY

Fuzzy logic is used in the operation or programming of air conditioners, cameras, digital image processing, elevators, microcontrollers, microprocessors, washing machines and many more.

We consider one such application : fuzzy logic in the working of air conditioners:
The main aim of an air conditioner is to regulate temperature and humidity. An air conditioning system is shown in figure below. There are two sensors in this system: one to monitor temperature and one to monitor humidity. There are three control elements: cooling valve, heating valve, and humidifying valve.



Air Conditioning System



Fuzzy Controller for Air Conditioning System

The following rules are taken into account:

- If temperature is low then open heating valve.
  For example, in the winter, when we use heat to raise temperature,

humidity is usually reduced. The air thus becomes too dry.

- If temperature is low then open humidifying valve slightly.
- If humidity is low then open humidifying valve slightly,

The air conditioner measures air temperature and then calculates the appropriate motor speed. The system uses rules that associate fuzzy sets of temperatures, such as ”cool,” to fuzzy sets of motor outputs,such as ”slow.” If a temperature of 68° Farenheit is 20% ”cool” and 70% ”just right” the system tries to run its motor speed that is 20% ”slow” and 70% ”medium”.

Thus, most modern systems in day to day life have some fuzzy aspect in it!

References

[1] George Bojadziev and Maria Bojadziev, *Fuzzy sets, Fuzzy Logic, Applications*,1998
[2] Petr Hájek, *Fuzzy Logic and Arithmetical Hierarchy*,1995
[3] *www.austinlinks.com/Fuzzy/tutorial.html*
[4] *http://plato.stanford.edu/*

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Extension of Course Contents

The paucity of time restricts us to our course content. Thus we wish to present this section which wil go beyond the scope of our text and will introduce concepts which are intriguing and also strengthen our knowledge and understanding.

Continuum Hypothesis

Abstract

This paper deals with the counting of the number of elements in an infinite set and the comparison of the infinities of different degrees.

Consider the following question:

If S_1 and S_2 are two circles having radii 1 and 2 units respectively, then which of the two has more number of points on its circumference?

For a layman, the answer may possibly be the bigger one, but for a person of mathematics the answer would be - both the circles have the same number of points! The argument for the assertion is simple. By drawing radii PP' and QQ' etc., we can pair up any point on the larger circle with a point on the smaller one and vice-versa. Hence the number of points (infinite) on the larger circle is same as the number of points (infinite) on the smaller one, though the length of the circumference of the large one is twice as that of the smaller one.

The same type of question can be asked about the set of natural numbers, set of even natural numbers, set of integers, set of rational numbers, set of real numbers etc. One may note that the problem posed above is in fact nothing but just that of *counting the size of infinite sets or comparing the size of different infinities.*

Counting the size of infinite sets was due until the late 19th century when a comprehensive theory of mathematical infinities was finally developed by George Cantor (1845-1918) in 1874.

In fact counting the number of elements in a set is precisely to determine its cardinality. Cardinality of a given set S means the number of element in it, denoted as $\#S$. If a set S contains n number of elements, then its cardinality is n, i.e., $\#S = n$, and the number n itself, is called the **finite cardinal number**. If a set contains an infinite number of elements, then its cardinality is called a **transfinite cardinal number**.

Based upon the number of elements, we divide the sets into two categories (i) finite sets and (ii) infinite sets. The latter can further be classified as denumerable and uncountable sets. To define these sets, we first define the equivalence of two sets.

Two sets A and B are said to be **equivalent** if there exists a 1-1 and onto correspondence between the sets A and B, and is denoted as $A \sim B$. More elaborately, two sets are equivalent if they have the same number of elements.

A set, say S, containing n elements, is said to be finite if there exists a 1-1 and onto correspondence between the set S and the set {1, 2, 3, ... , n}. Then we say that, $S \sim \{1, 2, 3, ..., n\}$, and $\#S = n$. In fact, the natural numbers along with singleton $\{0\}$, are sufficient to give the cardinality of any finite set.

Cantor defined an infinite set as the one which can be put into 1-1 and onto correspondence with a proper subset of itself. This had been proposed in 1872 by the German mathematician Richard Dedekind as - the whole is equivalent to a part of it. A classic example to understand it is that of the Hilbert's paradox of the Grand Hotel [5].

In fact, different characterizations of size, when extended to infinite sets, break various rules which hold for finite sets.

A set S is said to be **denumerable** if $S \sim \mathbb{N}$, and its cardinality is denoted by $\aleph_0$ (read as aleph nought), which in fact, is the first transfinite cardinal number, i.e., the cardinality of N is $\aleph_0$. The symbol $\aleph$ is the first letter of Hebrew script, and was given by Cantor. Here it is essential to understand that, $\aleph_0$ is just a symbol/thought, and saying a set to have cardinality $\aleph_0$ simply means that it has as many number of elements as the natural numbers.

An uncountable set means a non empty set which is not denumerable.

Now, with these definitions we are in a position to answer the question asked in the beginning.

The set of natural numbers $\mathbb{N}$ has the same number of elements as the set of even natural numbers $2\mathbb{N}$, since $\exists$ the map $f : \mathbb{N} \to 2\mathbb{N}$ defined by $f(n) = 2n$ which is 1-1 and onto, i.e. $\mathbb{N} \sim 2\mathbb{N}$ and hence $\#2\mathbb{N} = \aleph_0$. Likewise, the map $f : \mathbb{N} \to \mathbb{Z}$ defined by:

$$f(n) = \begin{cases} \frac{n}{2}, & \text{if } n \text{ is even} \\ -\frac{n-1}{2}, & \text{if } n \text{ is odd} \end{cases}$$

is 1-1 and onto, i.e.$\mathbb{N} \sim \mathbb{Z}$, and hence $\#\mathbb{Z} = \aleph_0$.

Now let us think about the cardinality of $\mathbb{Q}$, the set of rational numbers. Since we know that between any two rational numbers we can find an infinitely many rational numbers, one may feel tempted to know the count of $\mathbb{Q}$. Is it more than #$\mathbb{N}$? But, here comes another amazing fact that, $\mathbb{N} \sim \mathbb{Q}$ i.e., $\#\mathbb{Q} = \mathbb{N}$[2].

Now let us talk about the cardinality of $\mathbb{R}$, the set of real numbers. There are two possibilities, either $\mathbb{R} \sim \mathbb{N}$ or $\mathbb{R} \nsim \mathbb{N}$. The fact is that, there exists no 1-1 and onto correspondence between $\mathbb{R}$ and $\mathbb{N}$. The explanation for the same is not certainly direct. The idea is as follows.

The interval [0, 1] $\nsim \mathbb{N}$, i.e., the interval [0, 1] is uncountable[2]. Further, we have

1) [0, 1] $\sim$]0, 1[due to the function defined by

$$f(x) = \begin{cases} \frac{1}{2} & \text{if } x = 0 \\ \frac{1}{n+2} & \text{if } x = \frac{1}{n}, n \in N \\ x, & \text{if } x \neq, \frac{1}{n}, n \in N \end{cases}$$

2) [0, 1] $\sim \mathbb{R}$ due to the function defined by

$$f(x) = \begin{cases} \frac{2x-1}{x} & \text{if } 0 < x < \frac{1}{2} \\ \frac{2x-1}{1-x} & \text{if } \frac{1}{2} \leq x < 1 \end{cases}$$

The cardinality of [0, 1] is denoted by C. The letter C stands for continuum and refers to the Cantor's Continuum Hypothesis. Since [0, 1] $\sim$]0, 1[$\sim \mathbb{R}$, the cardinality of all these sets is C. Now before we write $\aleph_0 < C$, let us assign a meaning to the comparison of two cardinal numbers.

If A and B are any two sets, then we say that $\#A < \#B$ if A is equivalent to some proper subset of B but not equivalent to B.

Hence the cardinality of $\mathbb{N}$ is less than the cardinality of $\mathbb{R}$. Notice that though both the cardinalities are transfinite, they are comparable. i.e., we have two infinities, one bigger than the other in a certain sense. Having done so, let us consider the following two questions.

Question 1. Is there any transfinite cardinal number greater than C? Or, is there a largest transfinite cardinal number?

The answer to this question is given by the following fact:

For any set A, we always have $\#A < \#P(A)$, where $P(A)$ denotes the power set of A.[6]

Now consider the set $F = \{\text{collection of all functions } f : A \to B\}$ where A and B are any two non empty sets. If $\#A = \alpha$ and $\#B = \beta$, then $\#F = \beta^{\alpha}$ [3]. In particular, if we take $F' = \{\text{collection of all functions } f : A \to \{0,1\}\}$, then $\#F = 2^{\alpha}$.

Now P(A) $\sim F'$, since we have the function $\phi : P(A) \to F'$ defined by $\phi(B) = \chi_B$ for $B \in P(A)$, which is 1-1 and onto, where χ denotes the characteristic function.

Therefore, $\#P(A) = 2^{\alpha}$. Further, in particular, if we take A to be $\mathbb{N}$, then we have $\#P(\mathbb{N}) = 2^{\aleph_0}$. Hence $\aleph_0 < 2^{\aleph_0}$.

Now, let us re-denote $2^{\aleph_0}$ by $\aleph_1$. Then on taking $P(P(\mathbb{N}))$, we get $\aleph_1 < 2^{\aleph_1}$. Then re-denoting $2^{\aleph_1}$ by $\aleph_2$, and then taking $P(P(P(\mathbb{N})))$ we get $\aleph_2 < 2^{\aleph_2}$, and so on. Thus, we have a trail of transfinite cardinal numbers as below

$$\aleph_1 < \aleph_1 < \aleph_2 ....$$

Now, if we consider $F'' = \{\text{collection of all functions } f : \mathbb{N} \to \{0,1\}\}$, then $F'' \sim \mathbb{R}$ [4], i.e., $2^{\aleph_0} = C$.

Thus, the answer to Quesion 1 is complete.

The above question leads to another meaningful but difficult question.

Question 2. Is C the second transfinite cardinal number? Or, does there exist some transfinite cardinal number between $\aleph_0$ and C?

The answer to the above question is in fact the Cantor's Continuum Hypothesis.

The origin of this problem may be an analogous behaviour of finite cardinal numbers, where if $n > 1$, then there always exists a cardinal number between n and 2^n.

According to the Cantor's Continuum Hypothesis:

> there exists no transfinite cardinal number between $\aleph_0$ and C.

The hypothesis is still unsolved, and two of the answers available to this are the following:

Kurt Godel(1940): **It cannot be disproved**
Paul J. Cohen(1963): **It is un-decidable**

In fact, in 1963 it was shown that the continuum hypothesis is independent of the axioms of set theory in the same sense, as the Euclid's 5th postulate on parallel lines is independent of the other axioms of geometry.

References

[1] Thomas A. Garrity, *All the Mathematics You Missed*, Cambridge University Press, Cambridge, UK, 2002.
[2] Seymour Lipschitz, *Theory and Problems of Set Theory and Related Topics*, Schaum's Outline, Schaum Publishing Co., New York, 1964.
[3] P. K. Jain and V. P. Gupta, *Lebesgue Measure and Integration*, Wiley Eastern Limited, New Delhi, 1986.
[4] Elliott Mendelson, *Number Systems and the Foundations of Analysis*, Academic Press, New York, 1973.
[5] George Gammow, *One Two Three ... Infinity*, Bantam Edition, The Viking Press, Inc., New York, 1971.
[6] Robert R. Stoll, *Set Theory and Logic*, Eurasia Publishing House, New Delhi, 1976.

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Dihedral Groups

Abstract

This paper gives a brief insight into a very basic understanding of dihedral groups. Starting with the concept of symmetry, which includes rotational and reflectional symmetries, a formal definition of dihedral groups is coined (in terms of generators and relations). Further this paper discusses the notation, order and certain basic properties of dihedral groups with the help of detailed study of dihedral group D_8. This paper also reflects upon the presence of dihedral groups in our surroundings and their applications in various other disciplines.

INTRODUCTION

Symmetries and **Permutations** in nature and in mathematics can be described by an algebraic structure called a group. If F is a figure in the plane or in space, a symmetry of the figure F is a bijection $f : F \to F$ which preserves distances; that is, for all points $p, q \in F$,the distance from $f(p)$ to $f(q)$ must be the same as the distance from p to q. The **set of all symmetries** of a geometric figure forms a group under suitable composition.

A set of generators $\{g_1, g_2 ....g_n\}$ is a set of group elements such that possibly repeated application of the generators on themselves and each other is capable of producing all the elements in the group. A group generated by one element is called a **cyclic group** denoted as $G =< a >= \{a_n = e : n \in \mathbb{Z}^+\}$. For example, $G_3 =< a >= \{e, a, a^2\}$ where $a_3 = e$.

- **All cyclic groups just have rotational symmetry.**

Consider the case when a group G is generated by **two** elements x and y. Then group G is defined as $G = \{x, y : x^n = y^m = e\}$, where $n, m \in \mathbb{Z}$ and e is the identity of the group G. Such a description of a group in terms of its generators and their relations is called a **presentation** of G. Here G is an infinite **non-abelian group**. Similarly, consider the case when a finite non-abelian group G is generated by two elements x and y a and group G is defined as

$$G = \{x, y : x^n = e, y^2 = e; xy = yx^{-1}\}$$

The above **presentation** is that of a dihedral group where x denotes the rotation and y denotes the reflection.

Definition of a Dihedral Group

A dihedral group is a group of symmetries of a regular polygon including both rotations and reflections.

A regular polygon with n sides has $2n$ different symmetries: n rotational symmetries and n reflection symmetries.

The associated rotations and reflections make up the dihedral group D_{2n}.

Label the n vertices of an n-sided regular polygon as 1,2,3.....n. In general, if r be the rotation anticlockwise about the origin through $2\pi/n$ radian and s be the reflection about the line of symmetry through vertex 1 and the origin, then a dihedral group is written as:

$$D_{2n} =< r, s|r^n = s^2 = e, rs = sr^{-1} >$$

Notation

There are two competing notations for the dihedral group associated to a polygon with n sides. In geometry the dihedral group is denoted as D_n while in algebra the same group is denoted by D_{2n} to indicate the number of elements i.e. the order of dihedral groups = $2n$; where n = number of sides of a regular polygon. (Throughout this paper, the notation D_{2n} is used.)

Observe that given any vertex i $(0 \leq i \leq n-1)$, there is a symmetry which sends vertex 1 into position i. Since vertex 2 is adjacent to 1, vertex 2 must end up in position $i + 1$ or $i - 1$.Moreover, by following the 1^st symmetry by reflection about the line through vertex i and the centre of the n-gon, one can see that vertex 2 can be sent to either position $i + 1$ or $i - 1$ by some symmetry. Thus, the ordered pair of vertices $1, 2$ may be sent to $2n$ positions upon applying symmetry.

Thus there are exactly $2n$ symmetries of a regular n-gon.

These symmetries are:

- n rotations about the centre through $2\pi i/n$ radian, $0 \leq i \leq n - 1$
- n reflections through n lines of symmetry

When n is odd each symmetry line passes through a vertex and the mid-point of the opposite side When n is even there are $\frac{n}{2}$ lines of symmetry which pass through 2 opposite vertices and $\frac{n}{2}$ lines of symmetry which perpendicularly bisect 2 opposite sides. In either case there are n axes of symmetry altogether and $2n$ elements in the symmetry group.

The $2n$ elements of D_{2n} are $\quad e, r, r^2, ...r^{n-1}, s, rs, r^2s, ..., r^{n-1}s$

Clearly $r^n = e, s^2 = e$. Also, each element of the group has the form r^a or $r^a s$ where $0 \leq a \leq n-1$ And we can geometrically determine that, for $0 \leq a, b \leq n-1$

$$r^a r^b = r^k$$

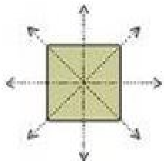
$$r^a(r^b s) = r^k s \qquad \text{where } k = a \oplus_n b \tag{1}$$

$$(r^a s)r^b = r^l s$$

$$(r^a s)(r^b s) = r^l \quad \text{where } l = a \oplus_n b \tag{2}$$

DIHEDRAL GROUP D_8

Let us consider the example of D_8 represented by a square object.



If we rotate and reflect the square in all possible ways about 4 (n=4,which is even) lines of symmetry (2 lines of symmetry which pass through two pair of opposite vertices and 2 which perpendicularly bisect the two pair of opposite sides), we observe that there are overall 8 possible ways of repositioning the square object. Therefore, the eight motions are:

R_0 = Rotation of 0°, R_{90} = Rotation of 90°, R_{180} = Rotation of 180°, R_{270} = Rotation of 270°
H=reflection about the horizontal axis, V=reflection about vertical axis , and D and D' represent reflection about the axes about the two diagonals.

We now claim that:

Any motion of the square, no matter how complicated is equivalent to one of these eight

Suppose a square is repositioned by a rotation of 90° followed by a reflection about the horizontal axis of symmetry. The final position so obtained is the same as the one obtained after applying the motion D. That is, we observe $HR_{90} = D$. This observation suggests that we can compose two motions to obtain a single motion (and hence the equations 1 and 2). The eight motions of the square object may be viewed as functions from the set of points making up the square to the square itself and we can combine them using function composition.

Therefore, the eight motions: $R_0, R_{90}, R_{180}, R_{270}, H, V, D$,and D' together with the operation composition, form a Dihedral Group of order 8. It is denoted by D_8.

PROPERTIES OF DIHEDRAL GROUPS

The following are the properties of dihedral groups D_{2n} defined as:

$$D_{2n} = \{r, s | r^n = e = s^2, rs = sr^{-1}\}$$

1) $1, r, r^2, ..., r^{n-1}$ are all distinct and $r^n = e$, so $|r| = n$.
2) $|s| = 2$
3) $s \neq r^i$ for any i
4) $rs = sr^{-1}$,this indicates that r and s do not commute so that D_{2n} is non-abelian. In fact, dihedral groups are the only finite groups generated by two elements of order 2, such that the two elements are distinct. However, for $n = 1$ and $n = 2$, $D_2 \approx \mathbb{Z}_2$ and $D_4 \approx \mathbb{Z}_2 \times \mathbb{Z}_2$ (Klien's Four-Group) are respectively are two examples of abelian dihedral groups. For all other values of $n (n \geq 3)$, D_{2n} is non-abelian.
5) For $n \geq 3$, the centre of dihedral group denoted as

$$Z(D_n) = \begin{cases} \{R_0, R_{180}\} & \text{when } n \text{ is even} \\ \{R_0\} & \text{when } n \text{ is odd} \end{cases}$$

   $Z(D_n)$ does not contain a reflection(can be proved geometrically). Also, when n is odd then D_n cannot have a 180° rotation.
6) The product of two rotations or two reflections is a rotation; the product of a rotation and a reflection is a reflection.

INFINITE DIHEDRAL GROUPS

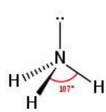
Consider the real line with the set of integers marked on it. Let G be the set of functions from the line to itself which preserve distance and which send integer among themselves. Then G is a group under composition of functions. Each element of G is a translation to the left or right through an integral distance, a reflection in an integer point, or a reflection in a point which lies mid-way between two integers. Let t be the translation to the right through one unit, so $t(x) = x + 1$ and let s be the reflection about the origin, so $s(x) = -x$. Then the elements of G are

$$...., t^{-2}, t^{-1}, e, t, t^2, ... \text{ and } ..., t^{-2}s, t^{-1}s, s, ts, t^2s, ...$$

where e is the identity function. For example, $ts(x) = -x + 1$ shows that ts is a reflection in the point $\frac{1}{2}$. The translation t and reflection s together generate G much like how reflection and rotation generate dihedral group D_n. Every dihedral group is generated by a rotation r and a reflection s; if the rotation is a rational multiple of a full rotation, then there is some integer n such that r^n is the identity, and we have a finite dihedral group of order $2n$. If the rotation is not a rational multiple of a full rotation and is replaced by a translation t of infinite order, then there is no such n and the resulting group has infinitely many elements and is called Dih_∞. It has the presentation $Dih_\infty = < r, s | s^2 = e, srs = r^{-1} >$ where r and s are usual notations for rotation and reflection.

DIHEDRAL GROUPS AROUND US

The dihedral groups arise frequently in art and nature. Many of the decorative designs used on floor coverings, pottery and buildings have one of the dihedral groups as a group of symmetry. Corporation logos are rich sources of dihedral symmetry. Chrysler's logo has D_{10} as a symmetry group, and Mercedes Benz has D_6. The ubiquitous five-pointed Red Star of David has a symmetry group D_{12}. Sea animals of the family that include starfish, sea cucumbers, feather stars, and sand dollars exhibit patterns with D_{10} symmetry group.



Chemists classify molecules according to their symmetry. Moreover, symmetry considerations are applied in orbital calculations, in determining energy levels of atoms and molecules and in the study of molecular vibrations. The symmetry group of a pyramidal molecule such as ammonia (NH_3), has symmetry group D_6. Mineralogists determine the internal structures of crystals (that is, rigid bodies in which the particles are arranged in three-dimensional repeating patterns - table salt and table sugar are two examples) by studying two dimensional X-ray projections of the atomic make up of the crystals. The symmetry present in the projections reveals the internal symmetry of the crystal themselves. Commonly occurring symmetry patterns are D_8 and D_{12}. Interestingly, it is mathematically impossible for a crystal to possess a D_{2n} symmetry pattern $n = 5$ or $n > 6$.

References

[1] Joseph A. Gallian, *Contemporary Abstract Algebra*, Fourth edition
[2] William J. Gilbert and W. Keith Nicholson, *Modern Algebra with Applications*, Second edition, pages 56-60
[3] M.A. Armstrong,*Groups and Symmetry*,1988 edition, page 23
[4] David S. Dummit and Richard M. Foote, *Abstract Algebra*, 3rd edition, Chapter 1
[5] *www.planetmath.org*

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Interdisplinary Aspects of Mathematics

Mathematics is everywhere around us. The applications of mathematics are diverse and innumerable. This section highlights such concepts of mathematics that are used extensively in day to day life. The practical aspect of mathematics, often overlooked, is what we seek to show in the following section.

Check Digits

Abstract

This paper introduces the concept of check-digits and throws light on its various applications. A check digit is the last digit of a barcode which comes into use to verify the accuracy of a barcode and to detect error while encoding. Barcodes are a part of our day-to-day lives, from the groceries to the pens, cars to your mobiles. Check digit on the barcode is the identification.

Introduction

A check digit is a digit attached to a number, to verify if the number is valid or not

A check digit, also known as a checksum character, is the number located on the far right side of a bar code. The purpose of a check digit is to verify that the information on the barcode has been entered correctly. The barcode reader's decoder calculates the checksum by performing a series of mathematical operations on the digits that precede the check digit, and comparing the result of the calculation to the value of the check digit. Typically, if the check digit matches the result of the calculation, the reader emits a signal (such as a beep) to acknowledge that the results match, and the scan has been successful It consists of a single digit computed from the other digits in the message.

With a check digit, one can detect simple errors in the input of a series of digits, such as a single mistyped digit, or the permutation of two successive digits. It is an easy way to encourage accuracy.

International Standard Book Numbers

The 10-digit ISBN format was developed by the International Organization for Standardization and was published in 1970 as international standard ISO 2108.

The International Standard Book Number identifies the country of publication, the publisher and the book itself. In fact, all relevant information in an ISBN is stored in the first nine digits which are then followed by the check digit.

If the digits of an ISBN are denoted $a_1, a_2, a_3......a_{10}$ with the first nine digits in the range 0-9, then a_{10} is chosen in the range 0-10 so that

$$a_1 + 2a_2 + 3a_3 + ... + 9a_9 + 10a_{10} \equiv 0 \mod 11$$

If a_{10} happens to be 10, it is recorded as an X.

For example, if we have the given ISBN 0-9553010-0-9

$$1(0) + 2(9) + 3(5) + 4(5) + 5(3) + 6(0) + 7(1) + 8(0) + 9(0) + 10(9)$$

$$= 165 \equiv 0 \mod 11$$

If the ISBN begins 0-93-603103, the tenth digit is chosen so that

$$1(0) + 2(9) + 3(3) + 4(6) + 5(0) + 6(3) + 7(1) + 8(0) + 9(3) + 10a_{10}$$

$$\equiv 0 \mod 11$$

Thus,

$$103 + 10a_{10} \equiv 0 \mod 11$$

so, $a_{10} = 4$
Therefore the check digit will be written as 4.

Now if this number was copied with an error, say in the fourth digit say as 0-93-503103-4,a computer could easily check that,

$$1(0) + 2(9) + 3(3) + 4(5) + 5(0) + 6(3) + 7(1) + 8(0) + 9(3) + 10(4)$$

$$= 139 \equiv 7 \not\equiv 0 (mod 11)$$

UNIVERSAL PRODUCT CODE

A universal product code is a way to represent a universal product number as a pattern of black and white stripes of various thicknesses. It is a number and bar code that identifies an individual consumer product. Most goods for sale today can be identified by its unique UPC number.
The universal product numbers are 12-digit numbers of the form x-xxxxx-xxxxx-x, where each x stands for a single digit between 0 and 9.the last digit here is the check digit. The check digit is calculated by the rule;

$$3(\text{sum of odd positioned digits}) + (\text{sum of even positioned digits})$$
$$\equiv 0 \mod 10$$

eg.

1 25002 74135 c

For the above barcode we have:
3(1+5+0+7+1+5) + (2+0+2+4+3+a12) = 0 mod 10 $\Rightarrow a_1 2 = 0 \Rightarrow 0$ is our check digit.

Error Correcting Using the Check Digit

Let a barcode be $U_1U_2U_3U_4U_5U_6U_7U_7U_8U_9U_{10}U_{11}U_{12}$ where each U_i, $i = 1, 2....12$ is the digit at the i^{th} place. We can always correct the error in case one of the bars is unreadable by the following formula:

$$X = Y - (W + Z)$$

where,
X = any single bar that is unreadable
W = $3(U_1 + U_3 + U_5 + U_7 + U_1 1) + (U_2 + ? + U_6 + U_8 + U_{10})$
Y = Integer divisible by 10 and greater than W
Z = U_{12} = Check Digit

For Example: Let a barcode be given where one of its bar is unreadable:

0-12?4567890-5

X = Y - (W + Z)
W = $3(0+2+4+6+8+0)+(1+?+5+7+9)=82$
X = W - (82 + Z)
Z = 5
X = W - (82 + 5)
X = W - 87
W = 90
X = 90 - 87
X = 3
Therefore adding 3 to the check digit will remove the error.

There is no doubt that the barcode technology has touched all our lives in one way or the other and its continuing to evolve as needs change. One has to admit that the digital revolution is fascinating and this technology will certainly become more refined and useful in the future.

References

[1] Edgar G. Goodaire and Michael M. Parmenter, *Discrete Mathematics with Graph Theory*, 2nd Edition, pages 135-136
[2] Joseph A. Gallian, *Contemporary Abstract Algebra*, 2nd Edition
[3] *T*he College Mathematics Journal

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Batting Average

Abstract

In this paper, I propose a method for computing a batsman's batting average that rectifies a major flaw in the current method of calculation.

INTRODUCTION

Cricket is a game played extensively in many countries. It is a game played between two teams, consisting of 11 players each. Ideally, a team consists of five batsman, one wicket-keeper and five bowlers. There are many known formats of the game, but in this paper, I shall concentrate on the 50 over format of the game called, One-day Internationals or ODIs. A match is broken up into two innings of 50 overs each. The innings ends when all the 50 overs have been bowled or if the 10 players have been out. There is always atleast one batsman who remains not out by the end of the innings.

THE CURRENT METHOD

A batsman's overall performance can be accessed by many statistical functions, the most important of them being, the batsman's average or his Batting Average. In simple terms, a batsman's average is the number of runs the batsman is expected to make in any particular innings.

$$\text{Batting Average is} = \frac{\text{number of runs scored by the batsman}}{\text{number of times he was out}}$$

The method of calculation is very similar to the calculation of the mean in statistics, except for one small difference. Suppose a batsman bats in x innings out of which he remained not out in p innings, and makes n runs in these x innings. Then his batting average is $\frac{n}{x-p}$, but his mean is $\frac{n}{x}$
The reason why the number of times the batsman remained not out is subtracted from the denominator is to give some credit to the batsman. For example, suppose in a match, Sachin scores 50* (not out) and Dravid scores 50, but gets out. To calculate their average, their total runs would be divided by their respective matches played. But Sachin would then not get any credit for remaining not out. If the match had continued, then he would have gone on to score more runs.

Thus, to give Sachin some credit, when his new average is calculated, 50 is added to his total runs, but 1 is not added to the denominator.

$$\underbrace{\frac{n+50}{x}}_{\text{Batting Average}} > \underbrace{\frac{n+50}{x+1}}_{\text{Mean}}$$

Thus, clearly this average would be greater than the mean that would have come out, hence giving Sachin some credit.

But there is a flaw in this method of calculation. There is always a possibility, especially for lower order batsmen that their batting average comes out to be greater than their highest runs ever scored. So in simple words, the batsman is then expected to make more than he has ever made before whenever he comes out to bat.

For example, in 2008, for the South African bowler, Dale Steyn, his batting statistics were

Matches	Innings	Not Outs	Runs	Highest Score	Batting Average
22	6	3	19	6	6.33

Clearly, his batting average is greater than his highest runs scored. This has often been seen for other lower order batsmen as well.

My Method

I present a new method for computing the batting average which removes the above flaw completely. For every innings in which a batsman is not out, I propose an algorithm to predict how many runs he would have scored, if he had continued to bat. The batsman would then be considered out at that score.

Our aim is to predict what he would have scored, if he had continued to bat. Suppose a batsman scores x_o^*(not out) when the match ends. Define a discrete random variable X taking values from $x_o.....x_t$ where x_t = highest runs ever scored by the batsman.

Define r_x to be the number of times he has made x runs
Define n to be the number of times he has scored $\geq x_o$
Define a function as:

$$f_X(x) = \begin{cases} \frac{r_x}{n} & \text{if } x \in x_o.....x_t \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

Then $f_X(x)$ is a probability mass function as:

1) By definition, the function takes positive values only

2) $\sum_{i=0}^{t} f_X x_i = \sum_{i=0}^{t} \frac{r_{x_i}}{n} = \frac{1}{n} \sum_{i=0}^{t} r_{x_i} = \frac{n}{n} = 1$

Now that we have a probability mass function, we need to find a mass point that suits our requirements. For that we find the median of this function. The Median M is defined as:

$$\sum_{x=x_o}^{M} f(x) \geq \frac{1}{2} \leq \sum_{x=M}^{x_t} f(x)\,, \text{ such that } \sum_{x=x_o}^{M-1} f(x) < \frac{1}{2}\,.$$

This M is our predicted score when the batsman scores x_o^*. An important thing to note here is that M is the batsman's hypothetical score. So, to calculate his average after this match, we add M to his previous hypothetical total and divide by the number of innings played. The batting average becomes the same as the batsman's mean, with the total runs being replaced the total hypothetical runs.

EXAMPLE

Let me explain this method with the help of an example. Suppose the batsman plays 10 matches and his scoring pattern is 5,7,3,8,9,4,0,5,4,8. Thus, his total runs are 53 and highest score is 9.

His batting average (according to old method) is same as the batsman's mean, since he has not been not out even once. His average is $\frac{53}{10} = 5.3$

In the 11th match, the batsman scores 5*. Now we will try to predict how much he would have scored if he had continued to bat, using his previous scores. Here:
x_o (his score in this match) = 5
n (total time he has scored more than or equal to x_o) = 6
x_t (his highest runs ever scored) = 9

The batman's distribution of scores is :

r_5	r_6	r_7	r_8	r_9
2	0	1	2	2

The median for this is 7, thus our predicted score is 7.
Now the average of the batsman after the 11th match is $\dfrac{53+7}{11} = 5.45$
With this method, Dale Steyn's average comes out to be 3.16, which is clearly less than his highest runs ever scored, i.e. 6.

Other batsmen's average, with this method comes out to be:

Batsman	Mean	Batting Average	My Average
Suresh Raina	28.0	34.0	30.2
Ishant Sharma	2.2	6.5	2.5
MS Dhoni	32.8	42.4	40.1
Michael Hussey	34.1	55.4	50.9
Yuvraj Singh	31.7	37.2	35.3

Thus we see that this method, like the current method, gives some credit to the batsman for remaining not out, but makes sure that the average would never exceed the batsman's highest runs ever scored, since the random variable does not exceed x_t.

I agree that this method of calculation is a lot more complicated than the current method, but with the advent of computers into statistics, anything can be calculated by just one click of the mouse. With a little more work on this method, I feel it could be used by the ICC to officially calculate the batsman's batting average.

References

[1] A.M. Mood, F.A. Graybill and D.C. Boes, *Introduction to the Theory of Statistics*
[2] *www.cricinfo.com*

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