[This question paper contains 6 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 8397 HC

Unique Paper Code : 32377908

Name of the Paper : Econometrics

Name of the Course : **STATISTICS : DSE for Honours**

Semester : V

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **five** questions in all.
3. Question **1** is compulsory.
4. Attempt **four** more questions, selecting **two** questions from each of the Section **A** and **B**.

1. (a) Fill in the blanks **(Attempt any five)** :

(i) Stimulus is another name given to ______ variable.

(ii) For $\underset{\sim}{Y} = X\beta + \underset{\sim}{U}$ where $E\left(\underset{\sim}{U}\,\underset{\sim}{U}'\right) = V$, the Generalized least squares estimator is given by ______ .

(iii) For including a categorical variable with 4 categories into the regression model, the number of dummy variables required is ______ .

(iv) A ______ model describes the complete structure of relationships among the economic variables.

(v) Bunch-map analysis is the modified version of ______ .

(vi) Rank correlation test is used to detect ______ .

(vii) Reduced form models express ______ variables as a function of ______ variables. (5)

(b) State whether True/False **(Attempt any five)** :

(i) White's test offers a solution to the problem of Heteroscedasticity.

(ii) Multicollinearity is a violation of the assumption pertaining to error terms.

(iii) The ordinary least squares procedure yields the BLUE of parameters of a General Linear model.

(iv) Weighted least squares estimator is same as Aitken estimator for a 2-variable regression model.

(v) Inclusion of irrelevant variables induces bias in the estimates of regression coefficients.

(vi) Dummy variables form a part of explanatory variables in a regression model.

(vii) In the model $y_t = \beta_2 x_{2t} + \beta_3 x_{3t} + \beta_4 x_{4t} + v_t$ where all the variables are deviations from their respective means, the degrees of freedom for sum of squares due to regression is 3. (5)

(c) Give short answers **(Attempt any two)** :

(i) Express the following model

Consumption function $C_t = \beta_0 + \beta_1 Y_t + U_t$; $0 < \beta_1 < 1$

Income identity $Y_t = C_t + I_t$

in the reduced form and interpret your result.

(ii) Consider the data

Y	-4	-2	0	2	4
X_2	1	2	3	4	5
X_3	3	5	7	9	11

(a) Can you estimate the parameters of the model, $Y_t = \beta_1 + \beta_2 x_{2i} + \beta_3 X_{3i} + U_i$. Justify your answer.

(b) If not, what linear functions of these parameters can you estimate? Show necessary calculations.

(iii) A study was performed to assess the impact of employment size (X) on the average compensation (Y) and following regression was obtained

$$\hat{Y}_i = 3417.833 + 148.767 X_i \qquad (*)$$

$$(81.136) \qquad (14.418)$$

$$\text{p-value} = (<0.001) \qquad (<0.001) \qquad R^2 = 0.9383$$

The analyst suspected violation in one of the assumptions and hence fitted the model

$$\frac{\hat{Y}_i}{\sigma_i} = 3406.639\frac{1}{\sigma_i} + 154.153\frac{X_i}{\sigma_i} \qquad (**)$$

$$(80.983) \qquad (16.959)$$

$$\text{p-value} = (<0.001) \qquad (<0.001) \qquad R^2 = 0.9993$$

(a) What violation is the analyst suspecting? Identify the specific form of this violation for which regression (**) provides a solution.

(b) What are the repercussions of such a violation? (2.5×2)

SECTION – A

2. (i) For the General linear model, show that the residual vector $\underset{\sim}{e} = M\underset{\sim}{U}$ where $M = (I_n - X(X'X)^{-1}X')$ and hence obtain an unbiased estimator of σ_u^2. Also find the Coefficient of Multiple determination. 5

(ii) For a General linear model, $\underset{n\times 1}{Y} = \underset{n\times k}{X}\ \underset{k\times 1}{\beta} + \underset{n\times 1}{U}$ with usual assumptions, develop the test of significance for individual regression coefficient. Also obtain 100(1-α)% confidence interval for β_i. (8,7)

3. (i) Discuss the Farrar-Glauber test for detecting Multicollinearity in a multiple linear regression model.

(ii) Explain the concept of Multicollinearity. Describe the terms "perfect Multicollinearity" and "high-but-imperfect Multicollinearity". What are the practical consequences of these? Illustrate your answer with the help of a suitable example. (7½,7½)

4. (i) In the case of General Linear model, obtain 100 (1 – α)% confidence interval for $E[Y_{n+1} \mid c]$ where $c = (1, X_{2.n+1}, \ldots, X_{k.n+1})$ the values of X in the period n + 1. Also find confidence interval for individual Y_{n+1}.

(ii) Suppose that we fit the model $Y_i = \alpha_1 + \alpha_2 X_{2i} + v_i$ instead of true model $Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i$. What kind of errors are observed in the resulting estimates. Derive expressions for those. (8,7)

SECTION – B

5. (i) For the 2-variable regression model, $Y_t = \beta_1 + \beta_2 X_{2t} + u_t$ where $u_t = \delta u_{t-1} + \epsilon_t$ where $E(\epsilon_t) = 0$; $v(\epsilon_t) = \sigma^2_\epsilon$ and $cov(\epsilon_t, \epsilon_s) = 0$ for $t \neq s$, derive the expression for

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the mean and variance-covariance matrix of $\underset{\sim}{U} = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix}$.

Hence obtain the expression for variance of β_2. Under what assumptions, does $V(\beta_2)$ in the above equals that of the OLS estimator. Suggest a suitable test to encounter the problem of autocorrelation.

(ii) Explain the Cochrane-Orcutt iterative procedure for a 2-variable regression model where errors follow AR(1) scheme. (10,5)

6. (i) Discuss the Goldfield-Quandt test for Heteroscedasticity.

(ii) In a 2-variable regression model $Y_i = \beta_1 + \beta_2 X_i + u_i$ where $E(u_i) = 0$, $V(u_i) = \sigma^2 X_i^2$ derive the expressions for the Generalized least square estimators along with its variance. Using $X_i = 1, 2, 3, 4, 5$ compare this variance with that of the variance of OLS estimator. (7,8)

7. (i) Describe the Koyck's approach for estimating the parameters of distributed-lag model.

(ii) Explain the concept of dummy variable regression models. Describe how the technique of dummy variables can be used for handling regression on one quantitative variable and one qualitative variable with more than two categories. (8,7)

(200)